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SCIENCE AND BROAD POINTS OF VIEW

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The conceptual revolution initiated by relativity and quantum theory, of which everyone has heard so much and with which everyone is so familiar, is, in my opinion, by no means yet complete but is still in the process of taking place, and the implications are not fully appreciated even by most scientists. In the following I should like to indicate, in the first place, some of the directions in which I think we may look for alterations or extensions of present points of view. After that, I shall try to suggest what some of the consequences of the altered points of view may be.

The conceptual revolution, as far as it has taken place up to the present, is often described as involving a realization that the traditional common-sense notions of daily life fail when they are extended into new realms of phenomena previously unknown. Thus we have the new realms of high velocities, with which we are concerned in relativity phenomena, or the realm of the very small, with which we are concerned in quantum phenomena. We have mostly met the situation up to the present by altering the meanings of many of our common-sense terms when applied in the new realms. For instance, one of the achievements of Einstein with his relativity theory was to alter the meaning of the term "simultaneity" when applied to systems in motion. But it is well recognized, on the other hand, that any altered meanings which are thus forced on us are ultimately altered meanings in the domain of everyday life. Strictly speaking, there is no such thing as a "microscopic world," for example; there is merely an altered macroscopic world, which we have found how to enrich and enlarge by manipulations which we formerly did not know how to make. The "microscopic world" which Leeuwenhoek was said to enter with his microscope was really a world of sensations with the same old sense organs which we have always had, and this world was "entered" by learning how to make new sorts of manipulations with the same old hands. The failure of the law of causality on the microscopic scale, to which quantum theory is said to force us, is really a failure on the scale of everyday life, for it is certain that we could couple a Geiger counter to an atomic bomb in such a way as to blow up the bomb under circumstances absolutely unpredictable, and it is this which principally concerns us. Or the unpredictable stars on photographic plates due to the disintegration of nuclei in cosmic radiation are phenomena on a scale of magnitude directly apprehended by our senses when we look at the plates.

It would appear, then, that the conceptual revolution is really on the level of ordinary experience with our senses. It is therefore natural to ask whether the discovery of new sorts of manipulation in the laboratory, the grinding of lenses to combine into a microscope, or the coupling of galvanometers and batteries in such a way as to respond to an atomic disintegration was really necessary to make us see that the nature of our common-sense concepts was not what we had supposed. Might it be that we could have anticipated some of the new points of view if we had only analyzed what we formerly knew with sufficient acuteness? I believe that this is the case. For instance, a realization that the classical Newtonian concept of simultaneity involved tacit assumptions about the nature of phenomena at high velocities, which might or might not be correct, could have been acquired by a sufficiently acute armchair analysis of the process of measuring the simultaneity of events, without waiting for the Michelson-Morley experiment. We can see this now, wise in hindsight. But, more than this, there are still many situations in daily life where we can see that, by pushing our analysis a little further, we can uncover things not yet thought of, or at least not yet assimilated into our thinking, and that, by so doing, we can anticipate further conceptual revolutions at least as great as those we have passed through. In fact, the common-sense way of conceptualizing can be touched at hardly any point without turning up things that, to say the least, are not appreciated or usually thought of. For instance, take such a universal and supposedly completely understood operation as counting. Detailed analysis will show that there are unexpressed and unrealized assumptions at the base of the everyday operation of counting which make it almost inevitable that the counting operation should be subject to limitations. "To count" is both a transitive and an intransitive verb. We can count things, or we can just count, without counting anything. Intransitive counting is typified by reciting to ourselves the succession of whole numbers. Even this is not simple. It requires the co-operation of memory, for we can "lose" our count. It requires some sort of pattern in the back of our heads. This pattern is so constructed that we can always take the next step, a simple situation which we forget when we allow ourselves to be bulldozed by the difficulties of the infinite. When we count *things*, we place the things in correspondence with the words we recite. We may do this by ticking off with our hand successive objects, keeping step with our recitation. But there is more to it than this. We have to be sure in some way that we are not counting the same thing twice. This demands certain properties in the objects that we count, and certain activities in ourselves to accompany our counting—we must be able to identify the objects and to remember whether we have counted them before or not. This does not usually seem to be appreciated. The situation covered by the concept "counting" is thus at least twofold: it involves something in us, and it involves something in the objects we count. If we had been alert enough to foresee that we might not necessarily be able to perform the necessary operations of identification on the "objects" we count, we might at least have been prepared for the possibility that electrons cannot be counted, a result of quantum theory which is often presented in an atmosphere of paradox.

If anyone questions the complexity of the counting operation, let him devise a counting machine which will count the number of *objects* of any sort in any arbitrary finite collection, not just count the eggs that file past on a belt conveyor. *Numbers*

do not occur in nature, nor does nature count. Wherever we find number, it is we who have put it there. One is reminded of the dictum of the mathematician Kroneker: "God made the integers—all else is man's work." It seems to me that we cannot grant the Diety even this minimum of function. Whenever we use numbers, we are engaged in something in which we ourselves are entangled, and what we find is in some way connected with ourselves. We would like to strip off the connection with ourselves and find the completely "objective" background; but the question is how to do it or even indeed, what it may mean.

Not only do numbers not occur in nature; neither do lengths or velocities or masses or energies or any of the other parameters with which we describe and explain the world about us. These occur not naked but only in conjunction with a human nervous system. An analysis which is pushed to the limit must say this out loud and not forget it, as we justifiably do for the purposes of daily life or for most of the purposes of physical science up to the present. We are just beginning to appreciate the complexities which we are neglecting when we disregard the fact that our whole knowledge of the world comes through the human nervous system and is formulated by that system. I personally experienced an unforgettably vivid appreciation of this on witnessing some of the demonstrations of the late Dr. Adelbert Ames, Jr., at Hanover, New Hampshire. These demonstrations have now been set up in more than one hundred research stations over the world, and the work is being actively carried on at Princeton by Professor Cantrill and his colleagues. Most of you are doubtless familiar with them. What I got out of them was an appreciation of the enormous complexity of the machinery that lies back of our perceptions and, most importantly, a realization that our world *is* the world of our perceptions. Our perceptions have a compulsive quality, even under conditions when we know that they cannot possibly be correct, as in many of the demonstrations, which can be appreciated only by experiencing them. The question could not but arise whether the world would have been the same if our nervous perceptual machinery had been different or, indeed, whether the fundamental forms of space and time, in terms of which we perceive the world, are good forms. They may be admitted to be good for ordinary experience; but are they good for the interior of the atomic nucleus or for the cosmological reaches of astronomical space and time, with their new potentialities for continuous creation? At the very least, one cannot help thinking that they may not be necessary forms of perception.

The upshot of what we have been saying is that the complete physical picture includes ourselves. Now physicists have been seeing this for some time and have developed methods of taking into account some of the consequences. It is widely recognized that many of the so-called "paradoxes" of quantum theory lose their paradoxical quality when the "observer" is included as part of the system, and one of the important problems of quantum theory is recognized to be to take proper account of this observer. This observer, if we are to treat the situation with full generality, is obviously ourselves, with our full complement of nervous system. It turns out, however, that for its own purposes quantum theory does not need to concern itself with so completely general an observer; it is adequate to consider only limited aspects. For quantum theory the "observer" is the measuring instrument, and the more obvious paradoxes can be fully explained by considering in

detail the reaction between the instrument of measurement and the object of measurement, a reaction which had been ignored in classical physics and in every earlier analysis of the physical situation. That this analysis goes far enough for many present purposes is proved by the many spectacular successes of quantum theory, but it obviously falls far short of an analysis that remembers that things like mass and energy occur only in conjunction with a human nervous system. If we could ever make such a searching analysis, I suppose that the most important question we would try to answer would be how to disentangle the human nervous system from the complex and arrive at something "objectively real." We are certainly very far from solving this problem now, to say nothing of whether the problem is solvable or even meaningful, but we can at least adumbrate here the possibility that our traditional analysis has overlooked something. What gives us pause is that a term such as "mass," which we apply to a simple elementary particle, occurs only when a nervous system, with its unfathomed complexity, is coupled to the particle. How necessary is the coupling? Have we added something of which we are unaware? Complete analysis would involve mastery of the nature of the nervous system. This is hopeless at present, but is it not possible that we could do something less than the maximum, which would at least relax the closeness of the coupling to our nervous system? There are interesting possibilities here that have not been worked out. At present the measurement of mass is a complicated process, involving calculation and the play back and forth of activities of the nervous system and instrumental activities in the laboratory. If we could devise an instrument which would indicate automatically on a scale the mass of a body to which it was coupled, we would at least have simplified the necessary co-operation of the human nervous system. We would then merely have to know that this particular instrument read "mass" and know how to use the number so obtained. Here is a new and inviting field, namely, the devising and analysis of instruments which automatically indicate one or the other of the conventional physical parameters when coupled to the object under investigation. I have made the beginning of such an attempt, but for our purposes it is not necessary to go into any detail. It is to be anticipated that the instruments will fall into hierarchies of greater or less complexity—that some instruments will be compound and incorporate other simpler ones and that when the use of these instruments is pushed to limiting conditions, some of the instruments will fail for larger-scale phenomena than others. Thus it will be found that an instrument for automatically recording mass is necessarily much more complicated than one that records position, for example, and that therefore, when pushed in the direction of small objects, the measurement of mass will fail before that of position. Now, if one *defines* mass as the number given by the mass-measuring instrument instead of as a number given by a human nervous system, then the *concept* of mass fails before that of position. Such an analysis has never been attempted, so far as I know. Until the analysis has been made, it is not clear whether such restrictions as the Heisenberg uncertainty principle are concealed here or not—probably not. But I think we may expect new things, and these may not be trivial.

There are at least two other directions in which an ultimately satisfactory analysis of the situation presented by quantum theory must go further than it does at present. One of these concerns the altered attitude toward *error* which quantum

theory apparently forces on us. For the notion of error is one of the apparently most fundamental and unavoidable in our dealing with the external world. We may recognize instrumental errors and errors which we ourselves make. The question of instrumental error does not concern us here; there are theories and methods of dealing with it which are roughly adequate. But errors made by ourselves are a more insidious matter. We know that we are prone to make them—we often have caught ourselves in an incorrect memory or even in writing a six with our hand when it was our firm intention to write a five. The possibility of this sort of error we always have with us. We can always ask ourselves whether we are making a mistake in reporting a given physical situation, and this is always a serious question to which we may not give a flippant answer. One of our most potent tools for dealing with this situation is repetition. If we repeat the experience which we are reporting and we can observe that our report is the same as before, we are likely to be satisfied that we have made no error. In fact, the repeatable situation is the cornerstone of modern science—the method of experiment on which most modern science rests would be irrelevant if experience were not repeatable. The demand that we have some method of assuring ourselves against self-error would seem to be pretty far-reaching. Such concepts as that of truth are involved, for what becomes of truth if the possibility of verification is absent? Or what shall we mean by the "reality" of the past if we cannot check the truth of our report of it? Fundamental modes of thought seem to be involved, but quantum theory appears to remove the cornerstone, for it insists that elementary situations in the microscopic domain are not repeatable. We may not, for example, aim an electron gun with such precision as to make a second electron hit a target in the same place as the first. The elementary act may not be repeated, may not be verified, and to that extent the concept of error is not applicable to it. But can I not always ask whether I made an error in reporting the impact of that electron?

We are apparently in the presence of a dilemma. I believe that the quantum point of view may here be more correct than that of common sense and that the way out of the dilemma may be in the direction already suggested, namely, in our insufficiently acute analysis of the situations of daily life. The seeds of the difficulty were perhaps already there before the new quantum effects were discovered. For I believe that events do occur in daily life of such elemental simplicity that the notion of error does not apply. It must in the first place be recognized that we do have adequate methods of dealing with the notion of error in many of the situations of daily life which are not repeatable. For there *are* situations in daily life which cannot be made to repeat. We may not repeat the summer shower to check whether we have correctly reported its occurrence. In such cases there are indirect methods of checking against error: thus we may observe that the street is wet, and this fortifies our belief that there was really a shower. Here we have an involvement of the shower with other phenomena, and we exploit the involvement to exorcise the possibility of error. But what about an event of such elemental simplicity that there is no involvement with other events, or at least no involvement that we can exploit? As an example, consider the flash of a firefly on a summer evening. This has no involvements that we can exploit, except under very special conditions. As it ordinarily occurs, the flash of the firefly can only be remembered and not checked in another way. If we ask how we can be sure that we have not an incorrect

memory of the flash, there is no answer. Nor can we question the experiencing of the memory. The memory of the unanalyzable event can only be accepted, and the notion of error does not apply to it. Neither does the notion of truth. A realization of this situation has not been adequately assimilated into our ordinary processes of thought, and new points of view are called for.

On the other hand, quantum theory has not been entirely successful in dealing with this situation either. There is a connection, although not a very close-knit one, between probability notions and notions of error. We deal with future events on the basis of probability: the future event can never be certainly predicted, but only its probability. The past event, on the other hand, can be stated to have occurred with certainty. This absolutely sharp break between the past and the future does not correspond to our actual experience—human uncertainty inheres in both the past and the future, and there is no sharp dividing line. There are other difficulties in this same area. Quantum theory has elevated probability from a derived notion, applicable in complex situations statistically as a veil for our ignorance, to a fundamental notion applicable to elementary situations. It is sometimes said in consequence that we can never have certain knowledge, but only probable knowledge. One asks, Probable knowledge of what? One can hardly ask the question or use the word "probability" in this context without the implication that there is something real in the background of which we have probable knowledge. But if the only knowledge is probable knowledge, it is meaningless to imply this real background. The difficulty may be to a large extent verbal, but even so it may be a fundamental difficulty, for we may have here one of those situations with which language is essentially incapable of dealing.

A second direction in which it is not unreasonable to anticipate that our future theory may be different from our present theory is with regard to the constants of nature which enter the fundamental equations. The equations of quantum theory contain two kinds of quantities. There are, in the first place, the several variable physical parameters which characterize the physical system, such as the positions and momenta of the various particles or the intensities of the field. These parameters are the result of measurement, and, as with all measured quantities, it is recognized that these are subject to uncertainty or mutual limitations and that an important task of present quantum theory is to analyze the interdependence of these limitations, as in the Heisenberg principle of uncertainty. The equations contain also a second class of quantity, exemplified by the charge on the electron or by Planck's constant. These, unlike position or momentum, are treated as absolute mathematical constants, the same in nature as π or $\sqrt{2}$. But of course they are actually experimental quantities, subject, like every other product of human frailty, to error. Quantum theory successfully ignores this feature. The reason that it can successfully ignore it must be that the way in which errors in position or momentum enter into the final mathematical edifice is essentially different from the way in which errors in the charge of the electron enter. But it can be only an incomplete analysis, justified, it may be, for particular purposes, which considers only the one kind of error and ignores the other. We may perhaps eventually expect a theory which more fully recognizes the entire gamut of human activity involved.

Our criticisms on the last few pages amount essentially to the criticism that the the human performer has not been adequately considered. One of the tasks of the science of the future is therefore a more adequate analysis of the observer, the observer being ourselves, as is. Now we are just beginning to see that this may be a task of unexpected difficulty and, indeed, that there may be essential limitations that will prevent us from ever doing as much as we would like. Of the importance of the task there can be no question; Socrates recognized it long ago with his admonition, "Know thyself." The detailed analysis of the nature of the observer and how to deal with him will doubtless be a long time coming, but one general feature of the situation can be formulated now. This is that the observer can never get away from himself or, more specifically, that we can never get away from ourselves. This insight is so obvious that it eludes us because of its very obviousness. Not only does it elude us because it is so close, but the insight cannot even be properly formulated in language. For when we try to say what it means to get away from ourselves, we find *ourselves* doing the saying or giving the meaning. We have here a situation that cannot be described except in negative terms; we are up against a fundamental limitation both of language and of thought itself. Now this business of trying to get away from itself is something that the human race has been trying to do for a long time, perhaps since the beginning of articulate thought. It colors our every attitude. It obtrudes itself whenever we talk or think about "reality." Perhaps the human activities in which the endeavor to get away from ourselves and to get into contact with an underlying reality is most articulate are philosophy and religion. In religion, particularly, man has from time immemorial been trying to get outside himself and find an absolute truth. But it is dawning on us that this is something that cannot be done.

What are the specific consequences of not being able to get away from ourselves? It is probably impossible to give a detailed answer at present, but it is possible to envisage at least one very general consequence. We are able to get this general glimpse because the consequences have been recently worked out in one special case. The special case is a situation in pure logic, and the detailed working-out of the answer has been accomplished by Gödel's theorem. Gödel was able to prove that within any logical system there are propositions which are unprovable within the system. In particular, it can never be proved by mathematics that mathematics is itself free from concealed inner contradiction. But this was exactly what some of the foremost mathematicians had been trying for a long time to do, and the proof that they were engaged on an intrinsically impossible quest, as impossible as squaring the circle or duplicating the cube, was terribly upsetting and is having revolutionary consequences in mathematics. One can formulate one of the consequences of Gödel's theorem by saying that the mathematician can never have certainty. The reason he cannot have certainty is that he cannot get away from himself. For, however far he may extend himself into metamathematics in his endeavor to prove the propositions of mathematics, it is still *his* metamathematics and part of *his* system.

The situations of logic are of childlike simplicity compared with the general situations presented by everyday language. One of the things that language does is to talk about talking; here we have a system trying to deal with itself, and we may ex-

pect difficulties. But more than this, and not the same thing, language talks about what it has just said. I can talk now about something that I and no one else could possibly have talked about one minute ago. Language is an enterprise that lays its own tracks as it proceeds. Not only this, but it seems to me that it often pulls up its tracks behind it, so that we find ourselves marooned in a wilderness of pure verbalism. The language of science, fortunately, does not have to meet these complexities.

In spite of the comparative simplicity of logic and its language, it took the highest technical competence to work out the consequences, and Gödel's accomplishment is still so fresh that the admiration and wonder are still with us. How much more in the more complicated situations of everyday language is it going to be difficult to work out the precise consequences, and how overwhelming the presumption must appear that revolutionary consequences and insights await us here. Although we may be far indeed from working out the detailed consequences, one consequence may be confidently assumed, namely, the consequence that certainty may not be attained and is a mirage. The insight that certainty does not occur is an insight that has taken Bertrand Russell a lifetime to acquire. From one point of view this insight is of the most banal obviousness. For we cannot imagine ourselves in a position in which self-doubt would not be possible, and there is no ultimate answer to self-doubt, Descartes to the contrary notwithstanding.

The problem of how to deal with the insight that we cannot get away from ourselves and that certainty does not occur is only part of a broader problem, the problem of understanding the nature of our minds, or the problem of understanding how we think and the limitations of thought.

One aspect of the problem of understanding better how we think is that of understanding better the structure and functioning of the brain with which we think. There has been much activity recently in this field, and we have a right to expect that ultimately a significantly better understanding will result. This activity is suggested by such catchwords as "cybernetics," "electroencephalograms," "communication theory," and "computing machines." We are learning how to construct complicated systems which successfully reproduce so many features of what is usually associated with thought that people seriously discuss the question whether these machines may properly be said to "think." The subject of whether human thought has characteristics which can never be reproduced by an admitted "machine" is still controversial. There are, however, certain very general points which seem to be generally admitted, which in themselves contain the seeds of revolutionary changes of attitude toward the thinking process. The enormous complexity of the brain is admitted, with the some ten billion neurons. These allow the possibility of so many different combinations as to provide fantastically more states of consciousness than a man could experience in his own lifetime or, for that matter, in the lifetime of the entire human race. That is, the brain as it is known today provides all the complexity that we can possibly need for a correlation with all possible mental experiences. By the principle of Occam's razor, we should not demand more or seek further until the present possibilities are exhausted. In other words, there is no reason at present to anticipate that an explanation of the processes of thought is not possible entirely in terms of the presently known naturalistic

processes of physics and chemistry. There is another consequence of the adequacy of an explanation of thought in terms of naturalistic brain processes. This is that it cannot possibly be that the brain deals with the external environment by any such device as constructing a replica of the external world, in which something in the brain stands in one-to-one correspondence with everything external. The reason is that the world is so extravagantly more complex than that part of it included in the brain that there simply are not enough atoms in the brain to make the correspondence. So long as it was thought that the processes of thought involved something analogous to a subtle, infinitely divisible continuous fluid, some such picture of mental action did not present internal inconsistencies, but with the now-accepted knowledge of the atomic structure of the brain and of the external world, such a view becomes impossible. It becomes evident that the brain is incapable of forming more than a very incomplete replica of the external world, and it becomes even more a problem than before to understand how we are able to adapt ourselves to our environment as successfully as we do.

The impact of our eventual better understanding of the nature of thought and its limitations is going to be much heavier outside the field of science than within it. There are at least two reasons. The first is that the situation has not been realized outside science to the extent that it is being realized within science, so that there is as yet little corresponding to the modest reformation in his thinking that the scientist has been making to meet the exigencies of relativity and quantum theory. The second reason is that the involvement of the processes of thought in the field of everyday activity is so enormously much more complex than it is in the field of science that the reformation will have to be more sweeping. As a single example justifying such a statement, consider human verbal behavior. This is incomparably more complex on the scale of daily life than it is in the laboratory. The scientist wants to make his verbal meanings precise and single-valued, and he has found a technique of doing this with tolerable success. In daily life, on the other hand, not only are meanings more complex, but it is even to be questioned whether the man in the street wants his meanings to be precise. The verbal behavior which has grown up to meet the situations of daily life is a terribly complicated thing, with a vast unconscious machinery of purposes and memories by which we communicate with our fellows, analogous to the complicated unconscious machinery by which we convert the primitive sensations of our sense organs into our perceptions of objects existing out there in time and space. In fact, I think it is suggestive to talk about a verbal perception analogous to our perception of objects.

It would seem that no field of human activity in which mental processes are involved will escape the impact of a better understanding of how to use our minds. One such very broad field is the field of the humanities. For it is obvious enough that there is a large intellectual component in any humanistic activity, whether or not the humanist would be willing to admit that his form of intellectual activity is closely akin to the intellectual activity of the scientist. This remains true even when we consider the one activity of the humanist which is supposed most definitely to set him apart from the scientist—his concern with values. For, although the ultimate acceptance or rejection of a value may be held to be determined by nonintellectual emotional factors, there is nevertheless a strong intellectual com-

ponent in any value situation, for values may be specified, appraised, and modified in the light of the consequences of acting according to them, and these are all activities of the intelligence.

This has a bearing on the supposed basic antagonism between the sciences and the humanities which has been so much accentuated in recent discussions. There is no sharp dividing line between the sciences and the humanities. Science is a human activity and to that extent shares features in common with the humanities. To the extent that this has been forgotten, the sciences are at present incomplete, and we have seen specific instances which call for modification and extension in our handling of scientific concepts. The humanities, on the other hand, have a very large intellectual component, using, for instance, the methods of logic in many situations, and to this extent make common ground with the sciences. To the extent that we use our minds, we are all in this thing together and have to understand how to use our minds if we hope to do more than just muddle through. In fact, I would make so bold as to maintain that it is even more important for the humanist to understand how to use his mind than it is for the scientist, because the situations which the humanist has to meet are so much more complicated than those of the scientist and because the unconscious assumptions which the humanist makes in handling his mind are so much more insidious and difficult to become aware of, because of their ubiquity, than the assumptions of the scientist. We have already spoken of the complexities of the speech of the humanist. These he can hardly avoid, since the overwhelming majority of the situations of concern to the humanist involve the use of speech to communicate with his fellows in society. There is an important feature here in this speech situation which we have not yet mentioned. This is that every individual of us is almost forced to use the speech of his fellows. He learns to use this speech as he uses a tool ready made, with no guaranty that the tool is well adapted to his own individual needs. The tool of speech which we all use is a tool developed by society through a long course of evolution, in which, of necessity, the long range and more or less permanent needs of society have exerted a much stronger formative influence than the more or less transient needs of the individual. Our present speech is so heavily weighted toward the needs of society that the individual often finds himself in situations in which he cannot express his needs or experiences with even simple integrity. For example, for the individual many values are variable, changing as he grows older, yet the implication of our ordinary value language is that values are constant and permanent. Society often intolerantly imposes on the individual values which he does not feel, with resultant unnecessary maladjustments.

The situations doubtless will have to be dealt with in stages. First must come an appreciation that the human race has not yet discovered how to use its mind. Practically everything I have said in this talk can be subsumed under this simple statement. We are getting at a realization of this from the side of science rather than from the side of the humanities, for reasons not reflecting on the humanist and already suggested. It must nevertheless be a self-confident humanist indeed who would not admit that it would be a miracle if forms of thought evolved by hit-or-miss methods to meet the needs of primitive societies should prove to be adequate in the incomparably more complex situations of modern society, particularly when the inadequacy has been proved in the simple situations of science. At least a

realization of this situation should be the possession of every member of modern society, and I, for myself, would not be willing to call a man properly educated who did not instinctively realize this, nor would I call any system of education adequate which did not impart this realization. The second stage, after the appreciation of the situation, is to devise methods of coping with it. This is, of course, a much more difficult matter, and something that we do not yet know how to do. Whatever the final answer, we can make one pronouncement now which we can anticipate that we will not have to take back. This is that the situation is *not* to be dealt with by succumbing to discouragement and turning back to the outlooks of the past. This procedure is, however, so often advocated that there is occasion here for active alarm. The situation that confronts us is a situation brand new in the history of the human race, both in our vision of the nature of the crisis and in our vision of possible ways of meeting it.

The scientist is in a position to play an important role here. In fact, I would be tempted to say that the scientist has a special responsibility, were it not that I regard the concept of responsibility as loaded and to be avoided. It must be admitted that the scientist, because of his special experience, is in a strategic position to see what the nature of the situation is. This is a simple statement of fact that may be made without immodesty, in the realization that the situations of the humanist are incomparably more complex than those of the scientist and that it is inevitable that the simple situations be apprehended first. What is more, the scientist, fortified by the spectacular success which he has admittedly attained in meeting the limited objectives of science, is in a unique position to confront the difficulties ahead without discouragement, confident of final success. The weariness and discouragement which too often impel the humanist to advocate a return to the past are symptoms of loss of intellectual morale. The modern world has lost its intellectual morale; the scientist can help to recover it. And it seems to me that a little more militancy on the part of the scientist would not be out of place.

THE ENZYMATIC ACTIVATION OF AMINO ACIDS VIA THEIR ACYL-ADENYLATE DERIVATIVES

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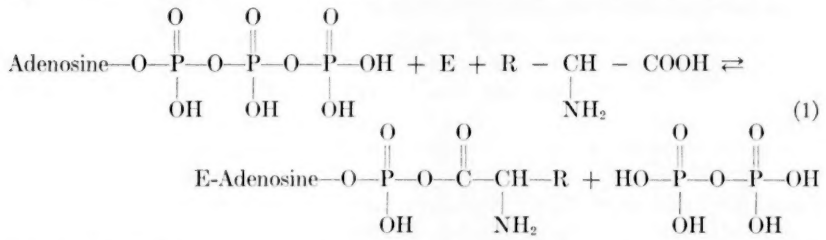
Although it has been known for a long time that energy is necessary for protein synthesis, there is little information available to suggest in which reaction energy is utilized for the eventual formation of the peptide bonds in the protein molecule. Recently several systems for the activation of amino acids have been described which may cast some light on the manner in which energy is utilized in protein synthesis.

An amino acid–dependent exchange between P^{32}-labeled inorganic pyrophosphate (PP) and adenosine triphosphate (ATP) was discovered by Hoagland[1] to be catalyzed by an extract of rat liver. We have found a similar enzyme system to be

distributed ubiquitously in microörganisms.[2, 3] Berg[4] reported the occurrence of a methionine-specific enzyme in yeast extracts which catalyzes the PP^{32}-ATP exchange reaction.

The reaction in each case appears to be a simple exchange on an enzyme surface, since careful balance studies have failed to indicate a net reaction. A clue to the nature of the reaction was given by the finding that the presence of high concentrations of hydroxylamine during the exchange led to a breakdown of ATP and the formation of PP, of adenosine monophosphate (AMP), and of the corresponding amino hydroxamates.[1] This observation suggested that amino acids are activated through their carboxyl groups during the exchange reaction. The fact that AMP does not undergo exchange under conditions where PP exchanges rapidly with ATP led Hoagland[5] to postulate the following mechanism for the exchange reaction:



(1)

It is visualized that the enzyme interacts with ATP and the amino acid, to split out PP and leave the intermediate, amino acyl–AMP, attached to the enzyme. This formulation is analogous to that postulated by Berg[4] for the activation of acetate by ATP in yeast extracts. In the latter case, acetyl-AMP was chemically synthesized and shown to be converted to ATP in the presence of PP or to acetyl-CoA in the presence of Coenzyme A. Equation (1) suggests that the carboxyl group of the amino acid undergoes reaction with ATP to form an anhydride with AMP through the elimination of PP. The amino acyl–AMP, being a substituted acyl phosphate, would be expected to be in the class of energy-rich compounds similar to acetyl phosphate,[6] acetyl-AMP,[4] and amino acyl phosphates[7] and might be expected to react with hydroxylamine at neutral pH to give the corresponding hydroxamate.

We have now obtained evidence in strong support of equation (1) by synthesizing L-leucyl-AMP (LAMP)[8] and demonstrating its ready conversion to ATP by a purified enzyme which catalyzes a leucine-dependent exchange between PP and ATP, apparently by reversal of reaction (1). The purpose of the present communication is to describe the synthesis of leucyl-AMP and its enzymatic conversion to ATP.

METHODS

The enzyme extracts used in these experiments were prepared by grinding cells of *Escherichia coli* with alumina and extracting with phosphate buffer, as previously described.[9] In certain cases the extracts were purified about 20-fold with respect to leucine-dependent exchange by treatment with ribonuclease, fractionation with ammonium sulfate, and adsorption and elution from calcium phosphate gel. Such preparations are free from inorganic pyrophosphatase and ATPase. The activity of the extracts was determined by measuring their ability to catalyze the amino

acid-dependent exchange between PP^{32} and ATP, as previously described.[9] An enzyme unit is defined as that amount which catalyzes the exchange of 1 μM of PP under defined conditions.[9] Radioactivity in ATP was measured by hydrolyzing ATP adsorbed to charcoal with 1.0 *N* HCl and counting an aliquot of the supernatant.[10] In some experiments, ATP was estimated as acid-labile phosphate[11] after elution from charcoal with 0.1 per cent ammonia in 50 per cent ethanol. In other experiments, ATP was measured by the decrease in acid-labile phosphate after incubation with an excess of yeast hexokinase and glucose. Inorganic pyrophosphate was measured by acid-labile phosphate or by the colorimetric method of Flynn *et al.*[12] Adenine was measured by optical density at 260 mμ and adenylic acid by using 5′ adenylic acid deaminase according to the method of Kalckar.[13]

Amino acyl–AMP was measured by the following modification of the original hydroxamic acid method of Lipmann and Tuttle.[14] The sample, containing 0.5–3.0 μM, is mixed with 1.0 ml. of 2 *M* hydroxylamine solution (prepared by mixing equal volumes of 4 *M* hydroxylamine hydrochloride and 3.5 ml. *N* NaOH). The volume of the reaction mixture is brought to 2.0 ml. with distilled water and allowed to stand at room temperature for 5 minutes. Then 2.0 ml. of a ferric chloride solution (1 part 3.5 *N* HCl plus 3 parts 20 per cent $FeCl_3 \cdot 6H_2O$ in 0.1 *N* HCl) are added. The reagents are mixed quickly and read immediately in a Klett colorimeter at 540 mμ. The readings must be taken quickly, since the color fades rapidly. Under these conditions 1.0 μM of leucine hydroxamate reads 130 Klett units. In some cases leucine hydroxamate was identified by one-dimensional paper chromatography using the solvent system 2-butanol:formic acid:H_2O (75:15:10). Leucine was estimated by the ninhydrin method of Troll and Cannon.[15]

Results

Synthesis of L-*Leucyl-AMP.*—Several methods for the preparation of the amino acyl–AMP compounds were first explored without success. Attempts to acylate AMP in aqueous pyridine by the method of Avison,[16] using N-carboxy amino acid anhydrides, apparently led to the formation of amino acid polymers. When the amino group is protected with an acetyl, carboethoxy, or carbobenzoxy group, AMP can be smoothly acylated by this method with a mixed anhydride prepared according to Wieland and Rueff.[17] However, all compounds prepared with a blocked amino group were inactive in our test system, and we have as yet been unable to remove the protecting group under conditions sufficiently mild to avoid destruction of the compounds. Since Berg[4] had successfully prepared acetyl-AMP by treating the silver salt of AMP with acetyl chloride, a method first introduced by Lipmann[18] in the synthesis of acetyl phosphate, we tried this method with the acid chlorides of amino acids and succeeded in preparing active compounds. The synthesis of L-leucyl-AMP is detailed below.

To 25.0 ml. of glacial acetic acid are added 2.0 mM of L-leucine acid chloride (HCl salt) prepared according to Levine[19] and 2.0 mM of disilver AMP; this material is shaken mechanically for 30 minutes at 30° C. The acid chloride is soluble in glacial acetic acid, while disilver AMP is not. At the end of the reaction time, 1.25 ml. of H_2O is added, and the suspension is allowed to stand at room temperature for 5 minutes. This procedure results in the decomposition of any unreacted acid chloride. During this time any free AMP, leucine hydrochloride, and leucyl-

AMP pass into solution in the glacial acetic acid. The remaining silver salt is removed by centrifugation. The supernatant is chilled to 0°, and 1 volume of cold absolute ethanol is added, followed by 2 volumes of cold ether. The resulting white precipitate is collected by centrifugation in the cold and then washed once with cold ether-ethanol (75:25), once with ether-ethanol (85:15), once with ether-ethanol (95:5), and finally with 100 per cent ether. The material is quickly dried by blowing a current of air into the tube. The product at this stage generally contains 7–8 μM of leucyl-AMP, approximately 13–15 μM of free AMP, and about 8 μM of free leucine per 10 mg. The over-all yield is about 9 per cent. We have found that this product, although contaminated with both AMP and free amino acid, is satisfactory for most purposes. We have also successfully prepared L-alanyl-AMP, D-leucyl-AMP, and DL-phenylalanyl-AMP by this method.

The compound may be further purified by passage through a Dowex column as follows: 50 μM of crude L-leucyl-AMP was dissolved in water and brought to pH 4.8 with solid $KHCO_3$. The solution was passed over a column of Dowex-1, X-10 (chloride form), 0.8 cm. in diameter and 4.0 cm. long. Leucyl-AMP passed through the column while free AMP was retained. This procedure resulted in the removal of almost all of the free AMP.

Characterization of Leucyl-AMP.—The proposed structure of the amino acyl–AMP compounds is presented in Figure 1. We have obtained evidence in support

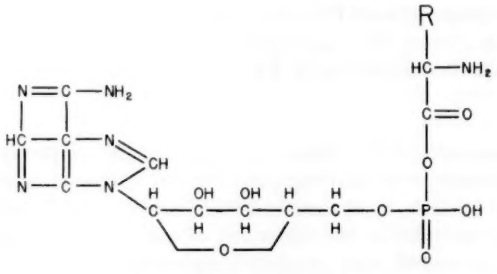


FIG. 1.—Amino Acyl-AMP

of this formulation in the case of synthetic leucyi-AMP. When the compound is treated with hydroxylamine in neutral solution, it rapidly reacts with hydroxylamine to give *only* leucine hydroxamate which has been estimated colorimetrically and identified by paper chromatography with an authentic sample of leucine hydroxamate prepared from the acid chloride. This is presumptive evidence that the carboxyl group of leucine exists in an anhydride linkage in the compound. The amount of adenine present in the compound was measured by its optical density at 260 mμ and expressed as 5′ adenylic acid. When a purified preparation of the compound is treated with 5′ adenylic acid deaminase, there is little or no change in optical density at 265 mμ, indicating that the 5′ adenylic acid in the compound is chemically bound in such a way that it was not attacked by the deaminase. When, however, a 5-fold excess of hydroxylamine is added to the cuvette containing the sample and deaminase, there is an immediate liberation of 5′-AMP, as evidenced by a rapid decrease in absorption at 265 mμ. Since hydroxylamine gives rise to

leucyl hydroxamate simultaneously with the liberation of 5′-AMP, it is clear that the carboxyl group of leucine is bound in an anhydride linkage with the phosphate group of AMP. A summary of these data is presented in Table 1.

TABLE 1

COMPOSITION OF LEUCYL-AMP

	μM*		μM*
Total adenine	1.06	Bound 5′-AMP	0.92
Free 5′-AMP	0.14	Leucyl hydroxamate	1.12

* For details of analysis see text.

By paper electrophoresis in 0.015 *M* potassium citrate buffer at pH 3.8 the compound was found to migrate with a positive charge, as would be expected from the structure given in Figure 1. Under these conditions, 5′-AMP migrated with a negative charge. All these facts are consistent with the formulation of the compound given in Figure 1.

Leucyl-AMP is quite stable at 37° in aqueous solution at acid pH. As the pH is increased, the rate of hydrolysis increases, and at pH 8.0 the compound has a half-life of about 4 minutes. The rate of hydrolysis at different pH values at 37° is plotted in Figure 2. The rate of hydrolysis appears to be increased by imidazole

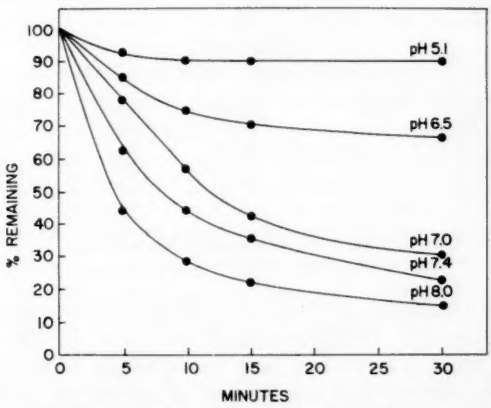


FIG. 2.—Effect of pH on rate of hydrolysis of Leucyl-AMP

uffer. For example, at pH 6.5 in phosphate buffer the half-life of the compound is greater than 30 minutes, whereas at the same pH in imidazole buffer the half-life is only 3 minutes.

Enzymatic Conversion of L-*Leucyl-AMP to ATP.*—In Table 2 are presented data which demonstrate the enzymatic conversion of L-leucyl-AMP to ATP. For routine analysis of reaction mixtures, it is more convenient to determine labeled ATP as counts per minute (c.p.m.) adsorbed to charcoal, as is routinely carried out in our exchange experiments. It was, therefore, important to compare the radioactivity method with more specific methods for measuring ATP. Accordingly, the reaction mixture described in Table 2 was divided into three aliquots, and ATP

was determined as counts per minute adsorbed to charcoal and compared with material eluted from the charcoal and assayed as acid-labile phosphate and as phosphate transferable to glucose by hexokinase. It can be seen that there is reasonably good agreement among the three methods. Therefore, radioactivity measurements were used in all subsequent experiments, thereby permitting considerable conservation of materials.

In the experiment described in Table 2, determination of residual leucyl-AMP indicated that the compound had largely disappeared in the control sample as well as in the complete system. These data were insufficient to make a balance study of the conversion of the compound to ATP. It was of interest, therefore, to compare the rate of disappearance of leucyl-AMP with the rate of formation of ATP. The results of such an experiment are presented in Table 3.

TABLE 2

FORMATION OF ATP FROM LEUCYL-AMP

	μM ATP DETERMINED BY		
	C.p.m. Adsorbed on Charcoal	Heat-labile P_i	Hexokinase Assay
Complete system	1.4	1.5	1.2
Enzyme omitted	0.03	0.01	..
LAMP omitted	0.07	0.02	..

* Conditions: 200 μM phosphate buffer, pH 6.8; 60μM $MgSO_4$; 20 μM PP^{32}; 6.8 μM LAMP; 0.4 ml. *E. coli* extract (200 units) in a final volume of 1.4 ml. Incubated 15 minutes at 37° C.

TABLE 3*

RATE OF FORMATION OF ATP FROM LAMP

TIME (Min.)	NO ENZYME		NO PP		COMPLETE	
	LAMP (μM)	ATP† (μM)	LAMP (μM)	ATP† (μM)	LAMP (μM)	ATP† (μM)
0	13.5	..	13.5	..	13.5	..
1	13.4	0.2	13.5	0.0	8.7	4.3
3	10.5	0.1	10.7	0.0	4.5	6.1
5	7.8	..	7.6	..	4.0	..

* Conditions: 300 μM phosphate buffer, pH 6.8; 90 μM $MgSO_4$; 30 μM PP^{32}; 13.5 μM LAMP; 0.8 ml. purified *E. coli* extract (600 units) in a final volume of 3.0 ml. Incubation was at 37° C.

† Determined by counts per minute adsorbed on charcoal.

In 1 minute of incubation at 37° C., 4.8 μM of compound disappeared and 4.3 μM of ATP was formed. After 3 minutes of incubation, 8.9 μM of compound had disappeared, of which 6.1 μM had been converted to ATP. During the same time interval the controls had lost 2.8 μM of compound, presumably through hydrolysis, since no ATP was formed. Thus, the leucyl-AMP which is not spontaneously hydrolyzed is quantitatively converted to ATP. The data of Table 3 also indicate that the conversion is dependent both upon the enzyme and upon the presence of inorganic pyrophosphate.

Since we had found previously[3] that the exchange reaction was specific for L-isomer of the amino acids and that, furthermore, crude extracts catalyzed the exchange in the presence of only eight active amino acids (leucine, isoleucine, valine, histidine, methionine, tryptophan, tyrosine, and phenylalanine), it became of interest to see whether the formation of ATP from the synthetic compounds had similar specificities.

In Table 4 are presented data showing that with a purified extract (purified about 20-fold with respect to leucine) only L-leucyl-AMP is active in forming ATP,

whereas D-leucyl-AMP has little or no activity. With L-alanyl-AMP, the ATP formed in the crude extract is significant, and this activity is lost as the enzyme is

TABLE 4*

SPECIFICITY OF THE CONVERSION OF LEUCYL-AMP TO ATP

ADDITIONS	μM ATP FORMED†			ADDITIONS	μM ATP FORMED†		
	No Extract	Crude Extract	Purified Extract		No Extract	Crude Extract	Purified Extract
None	0.1	0.1	0.1	D-Leucyl-AMP	..	..	0.3
L-Leucyl-AMP	..	1.5	1.3	L-Alanyl-AMP	..	0.6	0.2

* Conditions: 100 μM phosphate buffer, pH 6.8; 30 μM $MgSO_4$; 10 μM PP^{32}; 8 μM amino acyl-AMP; 200 units *E. coli* extract in a final volume of 1.0 ml. Incubated 5 minutes at 37° C.

† Determined by counts per minute adsorbed on charcoal.

purified. That fact that L-alanyl-AMP does form some ATP in the crude extract is of interest, since alanine is one of the amino acids which are inactive in the exchange reaction.

DISCUSSION

The fact that L-leucyl-AMP is converted to ATP by a purified extract which catalyzes a leucine-dependent exchange between PP^{32} and ATP and that the formation of ATP exhibits specificities similar to the exchange reaction suggests that amino acyl–AMP compounds are intermediates in the exchange reaction. Since no net breakdown of ATP can be observed during a reaction between ATP and leucine, even in the presence of a potent inorganic pyrophosphatase, whereas the intermediate compound, leucyl-AMP, is rapidly converted to ATP, it appears that the intermediate has little or no tendency to dissociate from the enzyme surface during the exchange reaction. All attempts to demonstrate net formation of leucyl-AMP, either isotopically with C^{14}-leucine or colorimetrically, gave negative results. Furthermore, no net reaction can be obtained from ATP and amino acid at pH levels where the intermediate would spontaneously hydrolyze rapidly, again indicating that essentially no dissociation of the intermediate occurs.

This situation is completely analogous to results obtained by Berg[20] with purified yeast aceto-CoA-kinase. In this case, also, exchange data indicate that acetate and ATP react with the enzyme to split out PP and that synthetic acetyl-AMP is converted rapidly to ATP in the presence of PP or to acetyl-CoA in the presence of CoA. However, here again all attempts to demonstrate the net formation of acetyl-AMP have proved negative.

The property of negligible dissociation would be beneficial in the case of the amino acyl derivatives, if these compounds are intermediates in protein synthesis, since the compounds would thus be protected from spontaneous breakdown or interaction until a natural acceptor was available to remove them from the enzyme surface. In the case of aceto-CoA-kinase, the natural acceptor of acetyl-AMP is CoA. It is reasonable to assume, therefore, that an analogous system may occur for the amino acyl–AMP compounds.

That the amino acyl–AMP compounds are indeed protected from spontaneous breakdown while they are on the enzyme surface is evident from the fact that in free solution the amino acyl–AMP compounds react immediately with hydroxylamine when the latter is present in low concentration, whereas a concentration of 1.0–2.0

M hydroxylamine is required to trap the activated amino acid during the exchange reaction.

The hydrolysis data indicate the great reactivity of these compounds, and since the breakdown of the compounds appears to be catalyzed by amino groups, it would appear that under appropriate conditions the amino acyl–AMP compounds could easily polymerize to form peptides or proteins.

Although we have not yet obtained direct evidence for the participation of this amino acid–activating system in protein synthesis, Hoagland *et al.*[5] have presented evidence that a similar system in rat liver may play a part in the incorporation of amino acids into microsomal proteins. Work is now in progress which is designed to obtain direct evidence for the participation of this activating system in protein synthesis. We are also attempting to determine whether the apparently inactive amino acids may be activated through a transacylation system with one of the active amino acyl–AMP compounds.

SUMMARY

The synthesis of certain amino acyl–AMP compounds has been described, and some of the properties of such compounds have been indicated. The rapid formation of ATP from leucyl-AMP and inorganic pyrophosphate by a purified extract of *E. coli* which catalyzes the leucine-dependent exchange of PP^{32} and ATP is reported. This finding suggests that amino acyl–AMP compounds are intermediates in the exchange reaction. The properties of these compounds as well as those of the enzyme system are discussed in relation to their possible participation in protein synthesis.

We are indebted to Dr. W. H. F. M. Mommaerts for a gift of 5′ adenylic acid deaminase.

* Dupont predoctoral fellow.

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[1] M. B. Hoagland, *Biochim. et biophys. acta*, **16**, 288–289, 1955.
[2] J. A. DeMoss and G. D. Novelli, *Bacteriol. Proc.*, p. 125, 1955.
[3] J. A. DeMoss and G. D. Novelli, *Biochim. et biophys. acta*, **18**, 592–593, 1955.
[4] P. Berg, *J. Am. Chem. Soc.*, **77**, 3163–3164, 1955.
[5] M. B. Hoagland, E. B. Keller, and P. C. Zamecnik, *J. Biol. Chem.*, **218**, 345–358, 1956.
[6] F. Lipmann, *Advances in Enzymology*, **6**, 231–267, 1946.
[7] A. Katchalsky and M. Paecht, *J. Am. Chem. Soc.*, **76**, 6042–6044, 1954.
[8] J. A. DeMoss, S. M. Genuth, and G. D. Novelli, *Federation Proc.*, **15**, 241, 1956.
[9] J. A. DeMoss and G. D. Novelli, *Biochim et biophys. acta* (in press).
[10] R. K. Crane and F. Lipmann, *J. Biol. Chem.*, **201**, 235–243, 1953.
[11] C. H. Fiske and Y. Subbarow, *J. Biol. Chem.*, **66**, 375–400, 1925.
[12] R. M. Flynn, M. E. Jones, and F. Lipmann, *J. Biol. Chem.*, **211**, 791–796, 1954.
[13] H. Kalckar, *J. Biol. Chem.*, **167**, 429–461, 1947.
[14] F. Lipmann and L. C. Tuttle, *J. Biol. Chem.*, **159**, 21–28, 1945.
[15] W. Troll and R. K. Cannon, *J. Biol. Chem.*, **200**, 803–811, 1953.
[16] A. W. D. Avison, *J. Chem. Soc.*, pp. 732–738, 1955.
[17] T. Wieland and L. Rueff, *Angew. Chem.*, **65**, 186–187, 1953.
[18] F. Lipmann and L. C. Tuttle, *J. Biol. Chem.*, **153**, 571–582, 1944.
[19] S. Levine, *J. Am. Chem. Soc.*, **76**, 1382, 1954.
[20] P. Berg, *J. Biol. Chem.* (in press) and personal communication.

*ENZYMATIC SYNTHESIS OF URIDINE DIPHOSPHATE XYLOSE AND URIDINE DIPHOSPHATE ARABINOSE**

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It has been shown previously[1] that mung bean seedlings contain a mixture of sugar nucleotides consisting of UDPG,[2] UDPGal, UDPXy, and UDPAr. In the present communication evidence is presented that these bean seedlings contain an enzymatic system capable of catalyzing the reversible formation of UDPXy and UDPAr from UTP and α-D-Xy-1-P.

When α-D-Xy-1-P and UTP were incubated in the presence of an enzyme preparation from mung beans, a UDPpentose was formed which proved to be a mixture of UDPXy and UDPAr. No arabinose derivative was formed in the absence of UTP. The formation of these products indicates that the preparation contains a uridyl transferase which forms UDPXy, and a waldenase responsible for the transformation of UDPXy to UDPAr.

The mung bean enzyme system was prepared as follows: 100 gm. of 5-day-old mung bean seedlings were blended in a Waring Blendor with 100 ml. of 0.01 *M* potassium phosphate buffer, pH 7.0; the suspension was strained through cheesecloth and centrifuged for 15 minutes at 11,000 × *g*. The supernatant solution was fractionated by addition of solid ammonium sulfate at 4° C. The fraction which precipitated between 55 and 60 per cent saturation was most active in catalyzing the formation of UDPpentose. This protein precipitate was dissolved in 2 ml. of 0.1 *M* Tris buffer, pH 7.5, and used in the preparation of the new sugar nucleotides.

A mixture of 13 μM of α-D-Xy-1-P, 25 μM of UTP, 2.5 mg. of inorganic pyrophosphatase,[3] 12.5 mμ $MgCl_2$, and 0.25 ml. of mung bean enzyme preparation (3 mg. protein) was incubated in a total volume of 1.2 ml. of 0.04 *M* Tris buffer, pH 7.5, for 90 minutes at 30° C.

The nucleotides in the reaction mixture were adsorbed on charcoal and eluted with ethanol-ammonia-water.[4] Dowex 50-H^+ was added to the eluate, and the suspension was filtered. The acidic solution containing the nucleotides was neutralized with sodium hydroxide to pH 7, and reduced in volume in a vacuum desiccator. The nucleotides were then separated by paper electrophoresis,[5] using potassium formate buffer, pH 3.5, with an ionic strength of 0.1. The separated nucleotides were eluted from the paper after being located with the aid of a Mineralight ultraviolet lamp. Determination of the optical densities of the eluted solutions at 260 mμ showed the formation of approximately 12 μM of UDPpentose. The reaction mixture also contained a small amount of residual UTP, together with UDP, UMP, and uridine, which were apparently formed by phosphatase present in the preparation. Analysis of the isolated UDPpentose is presented in Table 1.

The pentose sugars were identified as follows: Dowex 50-H^+ was added to an aliquot of the UDPpentose solution until the pH fell below 2. The solution

was decanted from the resin and heated at 100° C. for 15 minutes to hydrolyze off the pentose. The pentose was chromatographed two-dimensionally on paper, first with butanol–acetic acid–water and then with water-saturated phenol.[6] When the chromatogram was sprayed with aniline phthalate and heated, two red-colored spots appeared. The R_f values in both solvents of one spot corresponded to xylose, while those of the other corresponded to arabinose. Determination of the optical densities at 520 mμ of the solutions resulting from the elution of the spots with ethanol–hydrochloric acid–water[7] showed the pentose to be a mixture of approximately 30 per cent arabinose and 70 per cent xylose. The UDPpentose formed by mung bean preparations stored for 3 weeks at 2° C. contained only xylose.

TABLE 1

ANALYSIS OF UDPPENTOSE ENZYMATICALLY SYNTHESIZED FROM UTP AND α-D-XY-1-P

	μM PER μM URIDINE*	
	Found	Calculated
Total P†	2.12	2
Acid-labile P‡	0.96	1
Pentose§	0.99	1
Reducing value‖	0.06	0
Reducing value after hydrolysis#	0.98	1

* The concentration of uridine was determined by optical density at 260 mμ calculated on the basis of a molar extinction coefficient of 9.9×10^3.
† C. H. Fiske and Y. SubbaRow, *J. Biol. Chem.*, **66**, 375, 1925.
‡ Hydrolysis with 1 *N* H_2SO_4 for 10 minutes at 100° C.
§ W. Mejbaum, *Z. physiol. Chem.*, **258**, 117, 1939. The pentose value was corrected for the contribution of uridine.
‖ J. T. Park and M. J. Johnson, *J. Biol. Chem.*, **181**, 149, 1949. Reducing value calculated as xylose.
Hydrolysis with 0.01 *N* HCl at 100° C. for 15 minutes.

Because of the close configurational relationship between D-xylose and L-arabinose, it is reasonable to assume that the arabinose moiety of UDPAr is in the L-form. This is also the configuration of the arabinose in UDPAr isolated from mung bean seedlings.[8]

The UDPXy is apparently formed by a uridyl transferase which catalyzes the following reaction:

$$\text{UTP} + \alpha\text{-D-Xy-1-P} \rightleftharpoons \text{UDPXy} + \text{P-P}.$$

This type of reaction has been demonstrated with other sugar nucleotides.[9,10,11,12] β-D-Xy-1-P could not be substituted for α-D-Xy-1-P.

The production of inorganic pyrophosphate is adduced from the fact that a large accumulation of UDPpentose occurred only in the presence of inorganic pyrophosphatase. While UDPpentose was initially formed in the absence of pyrophosphatase, its concentration gradually diminished, presumably owing to the reversal of the reaction resulting from the gradual disappearance of UTP caused by phosphatase activity. The reversibility of this reaction was shown by the formation of UTP when isolated UDPpentose and inorganic pyrophosphate were incubated in the presence of the mung bean preparation.

UDPAr is apparently formed from UDPXy by a reaction analogous to that catalyzed by galactowaldenase,[7] which is responsible for the UDPG–UDPGal interconversion. The waldenase present in the mung bean inverts the OH group on the C-4 position of D-xylose, thus forming L-arabinose. This reaction is similar

to that of the galactowaldenase in which the OH group is inverted on the same position of D-glucose to form D-galactose.

It has not been determined whether this enzyme is a specific arabinowaldenase or whether it is the same waldenase that interconverts UDPG and UDPGal.

D-Xylose and L-arabinose are major constituents of various plant polysaccharides. Practically no information is available regarding the mechanism of synthesis of these pentose polymers. The discovery of pentose nucleotides in plants and the study of the various enzymatic reactions involved in their transformation may lead to an understanding of the mode of formation of these polymers.

Summary.—Mung bean seedlings contain a uridyl transferase which catalyzes the reversible reaction between UTP and α-D-Xy-1-P, forming UDPXy and presumably inorganic pyrophosphate. Another enzyme present in the mung beans, a waldenase, converts UDPXy to UDPAr.

We wish to express our appreciation to Dr. E. W. Putman for preparation of α-D-Xy-1-phosphate and β-D-Xy-1-phosphate and to Mrs. A. Munch-Petersen for her contribution of the inorganic pyrophosphatase.

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[1] V. Ginsburg, P. K. Stumpf, and W. Z. Hassid, *Federation Proc.*, **15**, 262, 1956.

[2] The following abbreviations were used: "UDPG" for uridine diphosphate glucose, "UDPGal" for uridine diphosphate galactose, "UDPXy" for uridine diphosphate xylose, "UDPAr" for uridine diphosphate arabinose, "α-D-Xy-1-P" for α-D-xylose-1-phosphate, "β-D-Xy-1-P" for β-D-xylose-1-phosphate, "UTP" for uridine triphosphate, "UDP" for uridine diphosphate, "UMP" for uridine monophosphate, "P-P" for pyrophosphate, and "Tris" for tris(hydroxymethyl)aminomethane.

[3] L. A. Heppel and R. J. Hilmoe, *J. Biol. Chem.*, **192**, 87, 1951.

[4] E. Cabib, L. F. Leloir, and C. E. Cardini, *J. Biol. Chem.*, **203**, 1055, 1953.

[5] A. M. Crestfield and F. W. Allen, *Anal. Chem.*, **27**, 422, 424, 1955.

[6] S. M. Partridge, *Biochem. J.*, **42**, 238, 1948.

[7] L. F. Leloir, *Arch. Biochem. and Biophys.*, **33**, 186, 1951.

[8] V. Ginsburg, P. K. Stumpf, and W. Z. Hassid (in preparation).

[9] A. Munch-Petersen, H. M. Kalckar, E. Cutolo, and E. E. B. Smith, *Nature*, **172**, 1036, 1953.

[10] H. M. Kalcker, B. Braganca, and A. Munch-Petersen, *Nature*, **172**, 1038, 1953.

[11] E. E. B. Smith, A. Munch-Petersen, and G. T. Mills, *Nature*, **172**, 1038, 1953.

[12] A. Munch-Petersen, *Arch. Biochem. and Biophys.*, **55**, 592, 1955.

*RULES FOR TESTING STABILITY OF A SELECTIVE POLYMORPHISM**

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1. *Introduction.*—Recently there has been increased interest in selective polymorphism, and many new examples have been found in diverse organisms. In some cases dozens of alleles or pseudoalleles are maintained in apparent equilibrium. This poses an interesting problem in the theory of population genetics: What are the conditions, necessary and sufficient, for the maintenance of stable genetic polymorphism by selection? A general answer must be very difficult, but I have obtained a solution subject to two restrictions: (1) random mating and (2) constant genotypic selective values.

2. *Single Locus with Multiple Alleles.*—Suppose that a given locus has n alleles, and let x_i be the frequency of allele A_i ($i = 1, 2, \ldots, n$). Let m_{ij} ($=m_{ji}$) be the fitness of the genotype A_iA_j, measured in Malthusian parameters. With a continuous model of generation time,

$$\frac{dx_i}{dt} = x_i a_i \qquad (i = 1, 2, \ldots, n), \quad (2.1)$$

where a_i is the *average excess* of the gene A_i defined by the relations

$$a_i = a_{i\cdot} - \bar{m},\ a_{i\cdot} = \sum_j m_{ij}x_j \qquad \text{and} \qquad \bar{m} = \sum_i\sum_j m_{ij}x_ix_j = \sum a_{i\cdot}x_i$$

With a discrete model the fitness of A_iA_j is measured by the selective value w_{ij}, and the formula corresponding to equation (2.1) becomes

$$\Delta x_i = \frac{x_i\bar{a}_i}{\bar{w}} \qquad (2.1')$$

where $\bar{a}_i$ is defined as a_i, except with w_{ij} substituted for m_{ij}.

With random mating, the rate of change in mean fitness becomes

$$\frac{d\bar{m}}{dt} = 2\Sigma x_i a_i^2 = V_a, \qquad (2.2)$$

where V_a is the genic (additive genetic) variance. Equation (2.2) is equivalent to Fisher's well-known fundamental theorem of natural selection, and for a discrete model it takes the following form:

$$\Delta\bar{w} = \frac{V_{\bar{a}}}{\bar{w}}. \qquad (2.2')$$

For details of these points see Crow and Kimura.[1] Since the right-hand sides of equations (2.2) and (2.2′) can never be negative, the mean fitness of the population must increase as long as the genic variance in fitness is not zero. Therefore, when all the alleles are in stable equilibrium, the mean fitness must be at a relative maximum. Thus our main task reduces to finding the condition for a relative maximum

with side condition: $\sum_{i=1}^{n} x_i = 1$. In the following treatment a letter a_{ij} $(= a_{ji})$ stands for the fitness of A_iA_j, denoting m_{ij} if the model is continuous and w_{ij} if the model is discrete.

To obtain the required condition for a maximum, we could use the standard method of Lagrange multipliers. But the following procedure is more straightforward. Let δx_i be any small displacement of the gene frequency x_i, and let $\delta\bar{a}$ be the corresponding change in the mean fitness. Then

$$\delta\bar{a} = \sum\sum a_{ij}(x_i + \delta x_i)(x_j + \delta x_j) - \sum\sum a_{ij}x_ix_j = 2\sum_{i=1}^{n} a_{i\cdot}\delta x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\delta x_i\delta x_j.$$

Since all the gene frequencies are not independent, we regard x_n as a function of the remaining x's. Substituting

$$\delta x_n = -\sum_{i=1}^{n-1} \delta x_i$$

in the above expression, we have

$$\delta\bar{a} = 2\sum_{i=1}^{n-1} (a_{i\cdot} - a_{n\cdot})\delta x_i + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} t_{ij}\delta x_i\delta x_j, \tag{2.3}$$

where

$$t_{ij} = a_{ij} - a_{in} - a_{jn} + a_{nn} \qquad (a_{ij} = a_{ji}) \tag{2.4}$$

At the point of maximum $\bar{a}$, the first term on the right-hand side of equation (2.3) must be zero for any small displacement of gene frequencies, which means that

$$a_{i\cdot} - a_{n\cdot} = 0 \quad (i = 1, \ldots, n-1).$$

This gives a set of simultaneous equations for equilibrium gene frequencies. Thus, if we denote the equilibrium frequency of the ith allele by $\hat{x}_i$, we get

$$\hat{x}_i = \frac{\Delta_i}{\sum_{i=1}^{n} \Delta_i}, \tag{2.5}$$

where Δ_i is the determinant made by substituting ones for all the elements of the ith column in the matrix $A = [a_{ij}]$ $(i, j = 1, \ldots, n)$.

With these values of $\hat{x}_i$'s, it is not difficult to show that $a_{j\cdot} = \Delta/\sum_{i=1}^{n} \Delta_i$ for all j's, where Δ is the determinant of A. Therefore, the mean fitness at equilibrium becomes

$$\bar{a} = \frac{\Delta}{\sum_{i=1}^{n} \Delta_i}, \tag{2.6}$$

and the gene frequencies do not change:

$$\frac{dx_i}{dt} = 0 \qquad \text{or} \qquad \Delta x_i = 0.$$

We first require that this equilibrium is nontrivial:

$$\hat{x}_i = \frac{\Delta_i}{\sum_{i=1}^{n} \Delta_i} > 0. \tag{2.7}$$

Next we require that, since a has a relative maximum at that point, the second term of equation (2.3) must be a negative-definite quadratic form, i.e.,

$$t_{11} < 0, \qquad \begin{vmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{vmatrix} > 0, \qquad \begin{vmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{vmatrix} < 0, \qquad \text{etc., up to order } (n-1). \tag{2.8}$$

The right-hand side of equation (2.4) may be written symbolically as $(i - n)(j - n)$, which upon expansion gives $(i, j) - (i, n) - (n, j) + (n, n)$, in which (i, j) means a_{ij}, and so on.

Let $T_n = [t_{ij}]$ be the matrix of the quadratic form; then it will be shown that its determinant $|T_n|$ is equal to the denominator in equation (2.7):

$$|T_n| = \sum_{i=1}^{n} \Delta_i.$$

However, relation (2.8) requires that

$$(-1)^{n-1} |T_n| > 0.$$

This means that

$$(-1)^{n-1}\Delta_i > 0, \qquad \text{in order that } \hat{x}_i > 0. \tag{2.9}$$

The above analysis shows that a point of stable equilibrium is, in Wright's terminology, a peak in n-dimensional adaptive space. Furthermore, it is seen that only one peak is allowed for all the n alleles to coexist.

Summing up, the necessary and sufficient conditions for the maintenance of all the n alleles in stable equilibrium are that the quadratic form T be negative definite and $(-1)^{n-1}\Delta_i > 0$ for all $i = 1, 2, \ldots, n$. To demonstrate the applicability of this rule, simple examples will be shown:

(i) In the case of two alleles ($n = 2$), say A_1 and A_2, we require that

$$\Delta_1 = \begin{vmatrix} 1 & a_{12} \\ 1 & a_{22} \end{vmatrix} = a_{22} - a_{12} < 0, \qquad \Delta_2 = \begin{vmatrix} a_{11} & 1 \\ a_{21} & 1 \end{vmatrix} = a_{11} - a_{21} < 0$$

and

$$t_{11} = (1 - 2)^2 = (1, 1) - 2(1, 2) + (2, 2) = a_{11} - 2a_{12} + a_{22} < 0.$$

These are equivalent to saying that the heterozygote must be of greater fitness than either homozygote—a well-known condition for stable equilibrium.

TABLE 1

	A_1	A_2	A_3
A_1	−0.1	0	0.15
A_2	0	0	0.1
A_3	0.15	0.1	−0.2

(ii) Given three alleles A_1, A_2, and A_3 and fitness of possible genotypes in the form of Table 1, what can we say about the stability? Table 1 represents the A matrix in which fitnesses are expressed in Malthusian parameters. Thus

$$\Delta_1 = \begin{vmatrix} 1 & 0 & 0.15 \\ 1 & 0 & 0.1 \\ 1 & 0.1 & -0.2 \end{vmatrix} = 0.0050 > 0, \qquad \Delta_2 = 0.0225 > 0, \qquad \Delta_3 = 0.01 > 0,$$

$$\Sigma\Delta_i = 0.0375.$$

Next,

$$t_{11} = (1-3)^2 = (1,1) - (1,3) - (3,1) + (3,3) = -0.1 - 2(0.15) + (-0.2)$$
$$= -0.6, \qquad t_{12} = t_{21} = -0.45, \qquad t_{22} = -0.4,$$

giving

$$t_{11} < 0, \quad \begin{vmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{vmatrix} = \begin{vmatrix} -0.6 & -0.45 \\ -0.45 & -0.4 \end{vmatrix} = 0.0375 > 0.$$

Thus all the conditions are fulfilled, and all three alleles are maintained in stable equilibrium with frequencies

$$\hat{x}_1 = \frac{0.0050}{0.0375} = {}^2/_{15}, \qquad \hat{x}_2 = {}^9/_{15}, \qquad \text{and } \hat{x}_3 = {}^4/_{15}.$$

At this state the mean fitness of the population is

$$\bar{a} = \frac{\Delta}{\Sigma\Delta_i} = \frac{(0.1)^3}{0.0375} = \frac{0.4}{15}.$$

It is interesting to note that, in the absence of A_3, the remaining two alleles A_1 and A_2 cannot be kept in stable equilibrium, since A_2 will replace A_1 quickly. This example might suggest that a stable equilibrium can exist only when each heterozygote is fitter than any homozygote. However, this is not the case, and the reader may easily verify this by substituting + 0.01 for 0 as the fitness of A_2A_3 and noting that all three alleles persist in stable equilibrium.

3. *Independent Multiple Loci.*—The procedure will be explained for the case of two loci. We designate the fitness of the individual with genotype $A_iA_jB_kB_l$ by a_{ijkl} $(= a_{jikl} = a_{ijlk})$ and define various averages, using the conventional notation of the analysis of variance. For example, $a_{ij\cdot\cdot} = \sum_{k,l} a_{ijkl}y_k y_l$, $a_{i\cdot\cdot\cdot} = \sum_{j,k,l} a_{ijkl}x_j y_k y_l$, etc., in which y_k is the frequency of the kth allele (B_k) in the second locus $(k = 1, 2, \ldots, m)$. It is possible to show, then, that the fundamental theorem still holds for this case, and we can solve the problem by maximizing the mean fitness. As in the case of one locus, we require first that $\hat{x}_i > 0$, $\hat{y}_k > 0$ for all i and k. Second, the matrix

$$T = \begin{bmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{bmatrix} \tag{3.1}$$

must be negative definite. Here T_{xx} is a $(n-1) \times (n-1)$ matrix whose (i, j)th element is $\hat{a}_{ij\cdot\cdot} - \hat{a}_{in\cdot\cdot} - \hat{a}_{nj\cdot\cdot} + \hat{a}_{nn\cdot\cdot}$, or, symbolically, $(i-n)(j-n)\cdot\cdot$ where the hat represents the value at equilibrium. T_{xy} is an $(n-1) \times (m-1)$ matrix whose (i, k)th element is $2(\hat{a}_{i\cdot k\cdot} - \hat{a}_{i\cdot m\cdot} - \hat{a}_{n\cdot k\cdot} + \hat{a}_{n\cdot m\cdot})$ or $2(i-n)\cdot(k-m)\cdot$, etc.

As a simple but an interesting application of the above rule, it is possible to show that, without having overdominance, epistasis alone cannot maintain a balanced polymorphism.

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[1] J. F. Crow and M. Kimura, "Some Genetic Problems in Natural Populations," in *Proc. Third Berkeley Symposium Math. Stat. and Probability*, 1956 (in press).

SUPPRESSION OF BUNT-RESISTANT GENE ACTION BY ENVIRONMENTAL CONDITIONS

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The influence of temperature on the degree of bunt expression of inoculated wheat has long been recognized in a general way for certain commercial varieties.[1-4] Only recently have isogenic lines been compared in their bunt response to low temperatures during early stages of development.[5-8] Most of these efforts were confined to a single isogenic pair, Baart-Baart 38. This paper reports extensive suppression of the action of the Martin gene for bunt resistance in six variety pairs, essentially isogenic except for this gene, caused by completion of the entire growth cycle at low temperatures and modified by light conditions. Under these circumstances, susceptible varieties are essentially 100 per cent bunted, and the corresponding members of the pairs, normally considered nearly 100 per cent resistant, become considerably bunted.

Material and Methods.—The wheat varieties employed represent important commercial varieties of California. During the last thirty years, Briggs and co-workers[9] have employed backcross breeding methods to develop isogenic lines differing from the susceptible varieties only by the M gene derived from the variety Martin, which confers practical immunity to race T-1 of *Tilletia caries* (DC) Tul., the only race of bunt prevalent in California. Studies[5] in these laboratories showed that exposure of susceptible Baart wheat plants from chlamydospore-inoculated seeds, germinated at *ca.* 10° C. and grown at this temperature for 10 days following emergence, produced plants at room temperature that were 100 per cent bunted. The isogenic resistant Baart 38 developed normal-appearing kernels, some of which, upon microscopic examination, contained hidden in their interior relatively small numbers of chlamydospores, many of which were immature. This is considered a partial suppression of resistance or a partial expression of susceptibility. Later studies reported production of bunted kernels by Baart 38 grown to maturity at low temperature,[5] by two resistant varieties[7] grown to anthesis at low temperature before transfer to warmer greenhouse conditions, and by five resistant varieties[7]

transplanted to the field after early growth at low temperatures. Hidden chlamydospores were found[10] in three susceptible varieties as well as in two resistant varieties.

In the present work the plants were grown from seed to maturity at low temperatures, and the results produced by two conditions of irradiation were compared. Parallel sets of seeds were planted in pots for growth under incandescent and fluorescent light conditions. Six pairs of wheat varieties (the number following the variety name indicates the year of introduction as a resistant variety) were used for both light conditions, and all were grown under nearly identical temperature conditions. In the series under fluorescent lights, three additional varieties were included. Seeds for all varieties except Bunyip were of pure lines, each grown from a single head. The incandescent series was grown on a bench 3 feet higher than the fluorescent series, in the same room, permitting very little fluorescent light to reach the incandescent series. The fluorescent light from white slim-line tubes was supplemented by four 100-watt incandescent lamps.

Seeds were planted (both inoculated and noninoculated groups) at 10° C. Seven days later the lights were turned on, first using 100-watt blue clear bulbs (160–200 foot-candles at top of plants) for 16 days for the incandescent series. For the fluorescent series, the intensity was adjusted to match that of the incandescent series, as measured with a Weston Model 756 illumination meter. Emergence from the vermiculite began 8 days after planting and continued during 1 week. The plants had good color, and the leaves emerged from their coleoptiles in a normal fashion. At the age of 20 days (from planting) most plants were 6 inches tall, those under blue incandescent lights being slightly taller and more slender than those under fluorescent lights. On December 4, intensities at the tops of plants were 210–300 foot-candles (depending upon the position) under fluorescent lights and 200–250 foot-candles under blue lights. Intensities were then increased, substituting white for blue bulbs in the incandescent series and adding more lights in the fluorescent series so that 500–700 foot-candles resulted. Temperatures rose during light periods (20 hours per day throughout the experiment) to 12° C. under incandescent lights and to 11° C. under fluorescent lights. The minimum during dark periods was 8° C. At 36 days, light intensities were increased to 1,000 foot-candles at the plant tops, and from this time lights were kept just above the tips of the plants, which were now 15 inches high (third leaf) under incandescent lights and 7–11 inches high under fluorescent lights. One fertilization with Hoagland's nutrient solution was applied. At 56 days the four 100-watt white incandescent lights supplementing the fluorescent lights were exchanged for similar 60-watt bulbs; the plants were now 15–22 inches high, compared to 20–26 inches under incandescent lights. As the plants became taller, the temperature at soil level became easier to maintain at $9^1/_2°$–$10^1/_2°$ C., but the parts of plants nearer the lights became warmer than this. Most plants were elongating rapidly at 64 days; the winter wheats Martin and Turkey 10015, however, did not elongate until after 110 days and headed last. At 141 days the temperature was raised to 15–16° C. to promote maturation of seeds.

Heading times and harvest results are presented in Table 1. Bunted kernels were detected by visual examination. Results of the six susceptible varieties were averaged for comparison with similar averages of their corresponding resistant

TABLE 1

EFFECT OF LIGHT QUALITY ON BUNT DEVELOPMENT IN SUSCEPTIBLE AND RESISTANT ISOGENIC WHEATS AT LOW TEMPERATURE

		Noninoculated				Inoculated						
		Total				Total						
Variety	Gene	No. Plants	No. Heads	Av. Height (Inches)	Heading Time (Days)	No. Plants	No. Heads	Av. Height (Inches)	Heading Time (Days)	Av. No. Bunted Kernels per Bunted Head	Percentage of Plants Bunted	No. Seeds Examined Microscopically*
Fluorescent lights:												
Baart	..	14†	14	44.0	110	13	14‡	34.0	106	12.6§	100	...
Baart 38	M	19†	19‡	42.0	104	16	16	39.0	102	5.9	63	2[||]
Big Club	..	15	0	21.0	102	17	18‡	31.0	98	21.0§	100	...
Big Club 43	M	20	20‡	33.4	107	20	33‡	28.0	114	1.8	20#	...
Bunyip	..	20†	20‡	36.4	94	21†	21‡	27.3	102	10.0§	77**	...
Bunyip 41	M	18	17‡	32.4	94	18	18	34.5	94	1.8	33	1
Onas	..	16†	16	34.0	102	17	34‡	16.0	110	2.9††	100	...
Onas 41	M	19	19‡	32.0	95	19	20‡	33.4	90	8.6††	26	...
Poso	..	14	15‡	34.0	103	16†	16‡	26.4	107	14.0	94	...
Poso 48	M	17	17‡	32.9	102	18	22‡	32.0	99	2.5††	44‡‡	...
Sonora	..	18	18‡	35.7	90	17†	17‡	31.6	77	13.4	100	...
Sonora 37	M	16	1	27.0	98	14†	14	37.4	90	5.5	43	4§§
Susceptible av.	..	16	14	34.2	100	17	20	27.7	100	12.3	95	...
Resistant av.	..	18	16	33.3	100	17	20	34.0	98	4.3	38	...
Hope	?	15†	15	38.0	85	16	23‡	25.0	102	5.7[|||]	75	...
Martin	M	18	18	44.0	165	18	18	42.0	154		0	...
Turkey 10015	XY	11	9	23.0	157	15	22	30.5	153	1.0##	7	...
Incandescent lights:												
Baart	..	11†	11	41.0	94	14	15‡	36.0	94	14.0§	93***	...
Baart 38	M	20†	20‡	38.0	90	16†	16	35.6	85	1.5††	38	10†††
Big Club	..	16†	16‡	32.0	90	15	16‡	28.4	85	25.0§	100	...
Big Club 43	M	20	20‡	29.0	77	21†	21‡	31.0	81	0.0	0	...
Bunyip	..	20†	20‡	36.0	77	24†	24‡	33.5	81	13.7§	100	...
Bunyip 41	M	18†	18‡	37.0	81	18†	18‡	37.0	85	0.4	11	...
Onas	..	16†	16	34.0	90	17	17‡	30.0	81	14.0§	77	...
Onas 41	M	18†	18	32.0	85	18	31	29.0	81	2.7	17	7‡‡‡
Poso	..	13	13‡	29.0	81	14†	14‡	29.0	81	12.0§	93	...
Poso 48	M	18†	18‡	33.0	69	18†	18	35.0	73	3.2	28	4
Sonora	..	21	15	29.5	81	18†	18‡	33.0	81	15.5§	78	...
Sonora 37	M	12	11‡	28.4	91	17†	17‡	37.0	90	4.2	29	...
Susceptible av.	..	16	15	33.6	85	17	17	33.3	84	15.7	90	...
Resistant av.	..	18	17	32.9	82	18	20	34.1	83	3.2	20	...

* The only whole kernels produced.
† No tillers.
‡ No whole kernels.
§ Bunt balls plump.
[||] On nonbunted plants.
Four heads on nonbunted plants were blasted.
** Eighteen per cent (4 of the 5 nonbunted plants) had blasted heads.
†† Bunt balls small.
‡‡ Two heads on nonbunted plants were blasted.
§§ Three on a nonbunted plant, one from a head with 7 bunt balls.
[|||] Bunt balls mixed, both small and large; many heads dwarfed; 4 were blasted.
Only 1 bunted kernel produced.
*** No chlamydospores could be found in the single nonbunted head.
††† Nine kernels were from nonbunted plants; one was next to a bunted kernel.
‡‡‡ From 4 nonbunted plants.

varieties having the Martin gene. None of the 31 seeds from inoculated plants of these six pairs showed microscopic evidence of chlamydospores.

Many plants that matured early were cut off above the first node and allowed to grow tillers to determine whether the mycelium of *T. caries* had remained alive in the basal portions of the plants and was capable of causing bunt in the heads of new tillers. Before the experiment ended, many had produced tillers 8–33 inches high, and some had borne heads, many of which were empty. In the fluorescent series, noninoculated Onas and Onas 41 and inoculated Onas 41 produced several whole kernels, and no bunt was observed. However, in the incandescent series, inoculated Baart, Onas, and Sonora each produced 6 bunted kernels; whole kernels developed on noninoculated Onas, Sonora 37, and Bunyip 41 and on inoculated Onas 41. Thus, as reported earlier for Baart[6] under other conditions, the fungus remained alive in the basal regions of several plants.

Comparison between Plants under Fluorescent versus Incandescent Lights.—In the inoculated group the susceptible varieties under fluorescent lights were slightly more heavily bunted and less vigorous than those under incandescent lights, and the resistant pairs were twice as heavily bunted, with more bunt balls per head under fluorescent lights. Also, this light condition produced 100 per cent bunt in four susceptible varieties, compared with only two such varieties under incandescent lights. Relatively few whole kernels were produced, but noninoculated Baart and Onas produced more such kernels under incandescent lights. Characteristics of the noninoculated controls, such as height, number of heads, and size of heads, were very similar under these two conditions.

The average heading times were unaffected by inoculation, but differences up to 13 days occurred in some varieties. The heading times for the plants under incandescent lights averaged consistently about 15 days less than for corresponding groups under fluorescent lights. On this basis higher bunt incidence was associated with slower plant development.

The 100 per cent bunt expression by four varieties under fluorescent light equals that previously obtained with Baart[5–7] under milder conditions. Among the resistant varieties, Baart 38 was more highly bunted than reported earlier,[5] and, in addition, every corresponding variety with the M gene was very appreciably bunted (only Martin remained free of bunt). On the average, all the varieties were more heavily bunted in these experiments than in those reported earlier,[7] in which plants were transferred to warmer temperatures during the latter part of their development. The varieties Baart,[6,7] Big Club,[7] and Bunyip[6] have expressed *ca.* 100 per cent bunt under several conditions. Not until this experiment were Onas and Sonora 100 per cent bunted. Expression of resistance is apparently modified by other associated genes from the susceptible parent variety. Such susceptible varieties differ among themselves in the degree of susceptibility exhibited under given conditions of temperature and light. It is reasonable to expect that proper conditions may be found to cause any variety to bunt heavily. Duration and timing, as well as degree of temperature exposure, are probably more important for some varieties than for others.

This experiment demonstrates that potential bunt resistance due to the Martin gene is suppressed when the wheat plants are grown, under the conditions of light employed above, at low temperatures during their entire life-history. This suppression of gene action was shown for six different wheats.

The three additional wheats that could be accommodated under fluorescent lights were carefully chosen: Hope, because it has long been considered susceptible to bunt if planted in the fall (hence subject to lower temperatures[4] during early growth) but resistant when planted in the spring; Turkey 10015, because, although considered resistant, it had been bunted to a low degree in other experiments,[7, 11] and Martin, because it is the source of the M gene of the resistant varieties and had not readily produced heads under artificial conditions. Hope was severely bunted (75 per cent), and Turkey 10015 was bunted to a low degree. Inoculated Martin plants had slightly fewer nodes per rachis on slightly shorter plants than was true for the noninoculated controls. Production of whole kernels was better on Martin than on any other variety (equal to Baart under incandescent lights). Martin grew as well as any, headed nicely, and produced 6 whole kernels per head; when inoculated, it produced only 1.8 whole kernels per head. Hope produced 3 when noninoculated and 0 when inoculated. Perhaps here is evidence of infection by *Tilletia* in Martin, even though final expression of bunt was repressed. We must search further for conditions that will permit bunt expression in Martin.

Schaller[12] found recently that three resistant varieties (Baart 46, Ramona 50, and White Federation 38) had appreciably depressed yields when grown in the field from seeds dusted with chlamydospores, indicating an effect of *T. caries* infection without expression of bunt. A suggestion of a similar effect is found in the work of Mouravieff,[3] who observed that apparently healthy heads on plants from seed of bunted heads were shorter and less fertile than controls. Our experiments are consistent with the idea that infection has its effects on wheat, even though bunt is not expressed, and that under proper conditions resistant varieties may demonstrate only a low degree[5] of resistance by formation of hidden chlamydospores or by mere blasting of heads rather than by bunt formation.

Few data in the literature indicate that 100 per cent expression of bunt has been obtained. It is very unusual in plant-breeding work, the highest seldom reaching 90–95 per cent. The earliest such report known to the writer was by Hungerford,[13] who studied the effects of temperature and soil-moisture conditions on incidence of bunt. He found 100 per cent bunt when soil at 32 per cent moisture was kept at 9–15° C. during germination of Jenkins Club, a fairly susceptible variety. Smith[4] reported 98 per cent bunt on this same wheat in the field and 100 per cent on Hope grown continuously at 9° C. in the greenhouse. Bunt incidence on Hope decreased to 2 per cent when the temperature was raised to 21° C. after emergence. Faris[2] reported bunt incidence by *T. laevis* of 96 per cent in the field and 100 per cent in the greenhouse when wheat was germinated at 10°–14° C. and grown through the early seedling stages at this temperature under controlled conditions before transplanting. His results were much lower with *T. tritici* (*caries*).

Many field experiments probably involve threshold conditions of temperature (and possibly light intensity or photoperiod) that do not quite permit 100 per cent expression of bunt in susceptible wheats or 0 per cent in resistant varieties. In an early study[14] of factors that modify the resistance of wheat to bunt, Briggs stated that some susceptible plants nearly always escape infection. In the light of results now available, it is probable that only on rare occasions would a susceptible seedling actually not be infected from the thorough dusting with chlamydospores

given such seeds before planting. It is more likely that all plants are infected and suffer some degree of depressed growth therefrom, but small differences in the development of individual plants determine whether or not bunt actually appears in the head. Briggs suggested[14] that genetically resistant plants occasionally might become infected. In these cases, environmental conditions might be such that certain threshold limits were exceeded to permit some degree of suppression of resistance in certain plants equipped genetically to be resistant under slightly different conditions. Briggs fully recognized the serious effects of environmental factors as the probable cause of variation in the percentage of bunt in rows from the same parent, grown in successive years. He pointed out the great difficulty in distinguishing clearly (through field experiments) between the actions of modifying factors and the effects of environment. It is reasonable to expect that the use of suitable constant environmental conditions would permit a quantitative genetic study of such factors. A difference of a few degrees determines[15] whether varieties of wheat are susceptible or resistant to *Puccinia graminis tritici*.

In field experiments plant breeders must grow relatively larger numbers of plants to obtain results of high statistical significance, because of considerable variation in smaller populations, such as rod rows of 30–70 plants. For instance, Briggs[16] used 2 or more rod rows to determine bunt incidence on hybrids and used 30 or more rows of the susceptible parent and 10 or more of the resistant parent. He usually found more than 50 per cent bunt difference between parents classified as resistant and those classified as susceptible. He reported bunt incidence in Baart over a period of 10 years ranging from 47 to 95 per cent, averaging 80 per cent. Briggs also found a low average of 2.2 per cent bunt in Turkey 1558, a wheat having the T gene for resistance and classified as a resistant variety. The strong influence of environment was pointed out in experiments recently reported by Briggs and Holton[11] on Turkey 10015 (which has two weak genes, X and Y) planted in two widely different locations. This variety produced 36 per cent bunt in Pullman, Washington, showing little resistance against race T-1 of *T. caries*. However, in Davis, California, this same variety averaged only 3.8 per cent bunt over 11 years, with a maximum of 8.8 per cent bunt. In the limited experiments in which we have used this variety,[7] it appears on the threshold of exhibiting susceptibility rather than resistance.

The effect of environmental factors such as temperature and light on expression of bunt must be effected through an influence on the chemical composition of the host plant, particularly in the young ovule, where chlamydospores normally form. Whether or not chemical conditions are favorable in the ovule apparently determines whether or not chlamydospores may be developed by the mycelium. Chemical thresholds of concentration or availability of growth factors necessary for differentiation of chlamydospores from the mycelium must determine whether relatively few or any chlamydospores develop or whether an abundance of them produces the well-known plump bunt balls. Earlier opinions considered mycelial growth to be reduced in the resistant plant by chemical influence;[17] Smith[4] suggested that temperature affects the internal composition of the plant favorably or unfavorably for fungal growth, to explain the results found with the variety Hope. Lewis[18] has outlined a balance hypothesis of parasitism, with which views expressed here agree. Studies on the nutrient requirements of *T. caries*[19, 20] have shown

the extreme dependence of mycelial growth rate on thiamine and the fact that this fungus is very sensitive to changes in the medium so far as certain combinations of amino acids and the contents of certain inorganic ions and sugars are concerned. However, no medium has been produced which offers a favorable condition for production of chlamydospores; probably the media lacked necessary nutrients found in the young ovule of susceptible wheat plants.

Smith's results[21] on the reaction of Martin wheat to three races of *T. caries* indicated differences in the development attained by the chlamydospores as well as in the size of the bunt balls. He interpreted this difference as a type of resistance when bunt balls are small and angular, rather than large and plump, and when chlamydospores are thin-walled, immature, and distorted in shape. Our results reported earlier,[5, 6, 7, 10] as well as current results on resistant varieties, may be interpreted similarly.

Plants grown at 10° C. produced few tillers, averaging from 0.12 to 0.36 per plant. Big Club (noninoculated), under fluorescent lights, produced many tillers but no heads. In general, the resistant group tillered more than the susceptible. The average number of nodes on each rachis was remarkably constant (around 12), regardless of inoculation, light condition, or bunt resistance. The effect of inoculation on height of plants was negligible except under fluorescent light, where it caused a 19 per cent average decrease for the susceptible varieties.

It is apparent that continual growth at 10° C. has promoted more intensive and more obvious bunt in susceptible varieties than was produced under conditions modified in later growth periods to favor more rapid growth of wheat. Four of the six varieties were 100 per cent bunted, and the other two were highly bunted; the blasted heads of Bunyip plants may have resulted from *Tilletia* infection. Blasting was common on some resistant varieties presumably affected by *Tilletia* inoculation and infection (Big Club 43 and Hope). More blasted heads were found in the series under fluorescent light; probably the average percentages of bunt reported for this series, since they are based entirely on visual observation of bunt balls and neglect blasting, do not indicate the full ravages of the parasite.

The doubling of bunt incidence in resistant varieties under fluorescent light compared to incidence under incandescent light indicates a strong possibility that quality of light (energy distribution among wave lengths) is a factor in the expression of resistance by the Martin gene. This seems likely because temperatures under the two light panels were identical within 1° C. most of the time and within 3° C. late in the experiment, and the light period was constant. Since the light meter is not equally sensitive to different regions of the spectrum, the over-all intensities of light from the two types of source, although registered as almost identical, may be quite different in different regions of the spectrum, even though small incandescent lights were added to the fluorescent group to supplement the red region. Light quality as a factor in the expression of bunt resistance has not been investigated before; we expect to pursue this further.

The controlled-temperature room in which the plants were grown was made available through the courtesy of Dr. L. L. Claypool, of the Pomology Department.

[1] O. Munerati, *Rend. Accad. Lincei*, **32**, 285, 1923.
[2] J. A. Faris, *Mycologia*, **16**, 259, 1924.

[3] V. P. Mouravieff, *Rev. Appl. Mycol.*, **7**, 773, 1928.
[4] W. K. Smith, *Phytopathology*, **22**, 615, 1932.
[5] R. B. Griffith, F. P. Zscheile, and J. W. Oswald, *Phytopathology*, **45**, 428, 1955.
[6] F. P. Zscheile, *Plant Physiol.*, **30**, 432, 1955.
[7] F. P. Zscheile, *Phytopathology*, **46**, 168, 1956.
[8] F. P. Zscheile, *ibid.*, p. 182.
[9] C. A. Suneson, *Hilgardia*, **17**, 501, 1947; F. N. Briggs and R. W. Allard, *Agron. J.*, **45**, 131, 1953.
[10] F. P. Zscheile and M. Anken, *Phytopathology*, **46**, 183, 1956
[11] F. N. Briggs and C. S. Holton, *Agron. J.*, **42**, 483, 1950.
[12] C. W. Schaller, personal communication.
[13] C. W. Hungerford, *Phytopathology*, **12**, 337, 1922.
[14] F. N. Briggs, *Hilgardia*, **4**, 175, 1929.
[15] K. R. Bromfield, *Phytopathology* (Abstr.), **44**, 482, 1954.
[16] F. N. Briggs, *J. Genetics*, **27**, 435, 1933.
[17] O. von Kirchner, *Fühling's landwirtsch. Ztg.*, **65**, 92, 1916.
[18] R. W. Lewis, *Am. Naturalist*, **87**, 273, 1953.
[19] F. P. Zscheile, *Phytopathology*, **41**, 1115, 1951.
[20] Hazel C. Murray and F. P. Zscheile, *Phytopathology*, 1956 (in press).
[21] W. K. Smith, *Phytopathology*, **22**, 847, 1932.

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*OPEN MAPPINGS OF COMPACT CONTINUA**

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The purpose of this paper is to collect together results concerning the existence of monotone open mappings[1] of a compact metric continuum X onto Y. In most of these theorems dim Y may be $\geq$ dim X. Proofs of these results will be forthcoming in a series of papers, principally in the *Transactions of the American Mathematical Society* and the *Annals of Mathematics*. Most of the theorems stated here have previously been announced by the author in various abstracts in the *Bulletin of the American Mathematical Society*.

THEOREM I. *Let $n \geq 2$. Let M be a triangulable n-manifold, with or without boundary, and let K be a compact subset of M with dim $K \leq (n-2)$. Let g be a monotone open mapping of K onto a set Y. Then there exists a monotone open mapping f of M onto a space containing Y such that f over K is g and $f(M-K)$ does not intersect Y and is homeomorphic to $M-K$.*

THEOREM II. *Let $n \geq 3$. Let M be an n-manifold, with or without boundary. Let g be a monotone mapping of M onto a space Y. Then there exists a monotone open mapping f of M onto Y.*

THEOREM III. *Let R^k denote the (closed) k-cell and S^k the k-sphere. Then, for any $n \geq 3$ and $m \geq 2$, there exist monotone mappings f of R^n onto R^m, g of S^n onto R^m, f' of R^n onto S^m, and g' of S^n onto S^m. Additionally, we may require that the inverse of each point under f or g is locally connected.*[2]

COROLLARY TO THEOREMS II AND III. *If for any $n \geq 3$ and $m \geq 2$, M is an n-cell or n-sphere and Y is an m-cell or m-sphere, then there exists a monotone open mapping of M onto Y.*

THEOREM IV. *Let $n > 2$. Let S be any continuum imbedded in a triangulable n-manifold, R, with or without boundary. Let k be any positive integer $< n$. Let ϵ be any positive number. Then there exist a continuum M in R and an open atomic*[3] *mapping f of M onto S such that* (1) *dim $M = k$;* (2) *for each $s \in S$, $f^{-1}(s)$ contains a k-cell; and* (3) *for $s \in S$, the Hausdorff distance between s and $f^{-1}(s)$ is $< \epsilon$.*

The final two theorems deal with monotone open mappings of X onto Y such that all the point-inverses under the mappings are homeomorphic to a continuum Z. In effect, these theorems give what might be called "pseudo-fiberings" of X by Z.

THEOREM V. *Let X be the 2-cell or the 2-sphere. Then there exists a monotone open mapping f of X onto itself such that, for each $x \in X$, $f^{-1}(x)$ is a pseudo-arc.*[4]

THEOREM VI. *Let X be the universal curve,*[5] *and let Y be any locally connected continuum. Then there exists a monotone open mapping of X onto Y such that, for each $y \in Y$, $f^{-1}(y)$ is homeomorphic to X.*

Remarks: The author has previously shown[6] that there exists a one-dimensional continuum M and a monotone open mapping of M onto the Hilbert cube. Either this result, Theorem IV, or Theorem VI taken together with Theorem I shows, as does the Corollary to Theorems II and III, that there exist open dimension-raising mappings of manifolds. Thus the following question of Steenrod listed by Eilenberg[7] is settled: Does there exist an interior map of a manifold on a space of higher dimension? Alexandroff[8, 9] has raised a number of questions about open dimension-raising mappings. He has referred[8] to the possible existence of open mappings of a cube onto a cube of higher dimension as the "central" or "most remarkable" problem in the area.[10] The Corollary to Theorems II and III above settles Alexandroff's questions about the mappings of cubes in essentially complete generality.

A further question of Alexandroff's concerning possible open dimension-raising mappings of a one-dimensional subset of the 2-sphere is settled affirmatively in a previous paper by the author.[11]

It follows as an almost immediate corollary of Theorem V that the following question raised by Knaster[12] is settled affirmatively: Does there exist a continuum K, irreducible between some two of its points, which is the sum of the elements of a continuous collection G of nondegenerate continua such that (1) G is an arc with respect to its elements as points and (2) all the elements of G are homeomorphic to each other? Let X be the 2-sphere, and let $f(X) = X$, with f as in Theorem V. Then, for any arc t in X, $f^{-1}(t)$ is such a continuum K, as is easily proved.

Each of the author's earlier result,[11] Theorem I, and the Corollary to Theorems II and III shows that results asserted by Rozanskaya[13] are not valid.

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[1] By a *mapping* we shall mean a continuous transformation. A mapping f of X onto Y is said to be *open* (or interior) if the image of each open set is open and is said to be *monotone* if, for each $y \in Y$, $f^{-1}(y)$ is connected. Homeomorphisms and the projection mappings of cells onto cells of lower dimension are well-known monotone open mappings.

[2] In a footnote in his paper "Duality Laws and Dimension" (in Russian), in *Comptes rendus du Premier Congrès des mathématiciens hungrois, 1950* [Budapest: Akadémiai Kiadó, 1952], pp. 329–357), Alexandroff states that Keldys has an argument for the existence of a monotone map of R^3 onto R^4.

[3] A mapping f of X onto Y is said to be *atomic*, provided that, for each continuum K of X, $K = f^{-1}(f(K))$.

[4] The pseudo arc may be characterized as a chainable, hereditarily indecomposable continuum.

[5] The universal curve is a one-dimensional analog of the Cantor middle-third set. It is locally connected, contains no local separating points, and no open subset of it can be imbedded in the plane. It may be constructed by "punching holes" out of a three-dimensional cube. First a middle-ninth is "punched out" in each of the three directions (of the various edges); then eight middle-ninths are "punched out" of what is left in each of the three directions; and so on. The common part is the universal curve.

[6] R. D. Anderson, "Monotone Interior Dimension-raising Mappings," *Duke Math. J.*, **19,** No. 2, 359–366, 1952.

[7] S. Eilenberg, "On the Problems of Topology," *Ann. Math.*, **50,** 247–260, 1949.

[8] P. S. Alexandroff, "The Present Status of the Theory of Dimension" (in Russian), *Uspekhi Mat. Nauk*, N.S., **6,** No. 5 (45), 43–68, 1951 (*Am. Math. Soc. Translations*, ser. **2, 1,** 1–26, 1955); "Duality Laws and Dimension" (in Russian), in *Comptes rendus du Premier Congrès de mathématiciens hungrois, 1950* (Budapest: Akadémiai Kiadó, 1952), pp. 329–357.

[9] I. A. Vainstein, "On a Problem of P. S. Alexandroff" (in Russian), *Doklady Akad. Navk S.S.S.R.*, N.S., **57,** 431–434, 1947.

[10] It is known that there exists no open mapping of either the 1-cube or the 2-cube onto a cube (or anything else) of higher dimension.

[11] R. D. Anderson, "On Monotone Interior Mappings in the Plane," *Trans. Am. Math. Soc.*, **72,** No. 2, 211–222, 1952.

[12] Bronislaw Knaster, "Un Continu irréducible à décomposition continue en tranches," *Fundamenta Math.*, **25,** 568–577, 1935.

[13] Yu. A. Rozanskaya, "Open Mappings and Dimension" (in Russian), *Uspekhi Mat. Nauk* N.S., **4,** No. 5 (33) 178–179, 1949

CONCERNING THE ACTION OF A FINITE GROUP

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The object of this note is to study principally the action of a finite group on a closed manifold.[1] By the pair (G, X) we denote a finite group G acting on a compact Hausdorff space X. We denote the quotient (orbit) space of (G, X) by X/G. A finite transformation group (G, X) will be termed admissible if it satisfies one of the following conditions:

1. X is a differentiable manifold on which G acts differentiably.
2. X is a differentiable manifold with boundary on which G acts differentiably.
3. X is a finite cell complex on which G acts cellwise (the set of points fixed under an element of G form a subcomplex).
4. X is cohomology locally connected for the prime p, and order $G = p^j$.

We always let r denote the order of G. In a forthcoming note[2] Lang has used our observations in indicating for finite transformation groups an approach involving L-series, quite analogous to Artin's treatment of extensions of number fields.

We consider the natural map

$$\pi\colon X \to X/G,$$

and we wish first to study

$$\pi^*\colon H^s(X/G;\ C) \to H^s(X;\ C)$$

for complex coefficients. If we define

$$I^s(X;\ C) = \{h^s | h^s \in H^s(X;\ C),\ g^*(h^s) = h^s,\ g \in G\},$$

then we state

THEOREM 1. *If (G, X) is a finite transformation group on a compact Hausdorff space, then*

$$\pi^*\colon H^s(X/G;\ C) \approx I^s(X;\ C).$$

This extends results of Eckmann[3] and of Liao.[4] Next, using admissible transformation groups, we state

THEOREM 2. *If (G, X) is an admissible group of transformations, then*

$$\chi(X/G) = \frac{\sum_G \chi(F_g)}{r},$$

where $F_g \subset X$ denotes the set of points in X fixed under the element g.

In case condition 4 is satisfied by (G, X), then the coefficients are to be taken mod p. This extends a result of Floyd[5] and is somewhat similar to a result of Tucker[6] in case conditions 1 and 3 are satisfied by (G, X). We can prove Theorem 2 in cases 1, 2, and 3 by

LEMMA 1. *If (G, X) is an admissible transformation group, then*

$$\chi(F_g) = L(g),$$

where $L(g)$ denotes the Lefschetz number of g.

Actually, in case condition 4 is satisfied by (G, X), we should add the hypothesis that $H^s(X;\ C)$ should be finitely generated and terminate finitely.[4] However, Theorem 2 does not depend, in case 4, on this, but follows inductively from Floyd's result for $G = Z_p$.[5]

We may ask whether a relation analogous to Theorem 2 holds for a continuous transformation group, and as an indication we note

THEOREM 3. *If a compact connected Abelian Lie group L operates continuously on a closed manifold M, then*

$$\chi(F) = \chi(M),$$

where $F \subset M$ is the set of points fixed under the group L.

We might briefly mention that there is a form of Theorem 1 in case of integer coefficients. We again consider $I^s(X;\ Z)$ defined analogously, and for any finite transformation group on a compact Hausdorff space we have

$$\pi^*\colon H^s(X/G;\ Z) \to I^s(X;\ Z);$$

one may show that the kernel and the co-kernel of this map are both nilpotent Abelian groups for which the order of any element divides some power of r, the order of G.

In closing, we express our appreciation to E. Artin for his help and encouragement in preparing this note.

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[1] D. Montgomery and L. Zippin, *Topological Transformation Groups* (New York: Interscience Publishers, 1955), p. 40.

[2] S. Lang, "The L-Series of a Covering," these PROCEEDINGS (to appear).

[3] B. Eckmann, "Coverings and Betti Numbers," *Bull. Am. Math. Soc.*, **55**, 95–101, 1949.

[4] E. E. Floyd, "On Related Periodic Maps," *Am. J. Math.*, **74**, 547–554, 1952.

[5] E. E. Floyd, "On Periodic Maps and the Euler Characteristic of Associated Spaces," *Trans. Am. Math. Soc.*, **72**, 138–147, 1952.

[6] A. W. Tucker, "Branched and Folded Coverings," *Bull. Am. Math. Soc.*, **42**, 859–862, 1936.

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*A DIFFUSION EQUIVALENT TO A COUNTABLE MARKOV CHAIN**

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1. *Introduction.*—Let $K = [0, 1]$, let $C(K)$ be the space of continuous functions on K to the real numbers, let $m(dx)$ be a strictly positive Borel measure on K, and let B be the second-order differential operator $D_m D_x$. Subject to suitable boundary conditions, the problem

$$(\mu - B)w = v \qquad \mu > 0,\ v \in C(K), \tag{1}$$

has precisely one solution:

$$w = (G_\mu v)(x) = \int_K G(\mu, x, s)v(s)m(ds), \tag{2}$$

the kernel $G(\mu, \cdot, \cdot)$ being the appropriate Green function. One has

$$w = \int_0^{+\infty} e^{-\mu t}\, u(t, x)\, dt, \tag{3}$$

where $u(t, x)$ is the solution to the corresponding parabolic equation,

$$u_t(t, x) = Bu(t, x) \qquad u(0+, x) = v(x), \tag{4}$$

which satisfies the same boundary conditions. Also, this solution $u(t, x)$ has the representation,

$$u(t, x) = (T_t v)(x) = \int_K P(t, x, ds)v(s), \tag{5}$$

in which the operators $(T_t:\ t > 0)$ constitute a semigroup mapping $C(K)$ into $C(K)$ and the kernel $P(\cdot, \cdot, \cdot)$ satisfies the Chapman-Kolmogorov identity

$$P(t_1 + t_2, x, M) = \int_K P(t_1, x, ds)P(t_2, s, M), \tag{6}$$

each $P(t, x, \cdot)$ being a Borel probability measure on K, and one can construct a Markov process (diffusion) with sample functions x_t and transition probabilities,[1]

$$PR(x_{t+s} \in M \,|\, x_s = x) = P(t, x, M). \tag{7}$$

Here we shall suppose that $m(dx)$ *is concentrated on the rational numbers* $R = (r_i:\ i > 0)$ *in* $(0, 1)$, the mass $m_i = m(r_i)$ being >0,[2] *so that equation* (2) *becomes*

$$(G_\mu v)(r_i) = \sum_{k>0} G(\mu, r_i, r_k)v(r_k)m_k. \tag{8}$$

This makes one suspect that equation (5) *is about the same as*

$$(T_t v)(r_i) = \sum_{k>0} P(t, r_i, r_k)v(r_k) \tag{9}$$

and that the Markov process, advertised above, is (*roughly speaking*) *nothing but a countable Markov chain on R.*

To support these contentions, we shall show that

$$P(t, x, K - R) = PR(x_t \in R \,|\, x_0 = x) = 0 \qquad t > 0,\ x \in K, \tag{10}$$

which makes it clear that relation (9) is true and that the matrix $p(t)$, whose entries are $p_{ik}(t) = P(t, r_i, r_k)$, has row sums 1 and satisfies the Chapman-Kolmogorov identity

$$p(t_1 + t_2) = p(t_1)p(t_2) \qquad t_1, t_2 > 0. \tag{11}$$

This shows that the irrational numbers have no importance and that one can lump them into a single state, setting

$$\begin{aligned} \hat{x}_t &= i \qquad x_t = r_i \qquad i > 0 \\ &= +\infty \quad x_t \in K - R, \end{aligned} \tag{12}$$

this being a Markov chain on R with stationary transition probabilities,

$$PR(\hat{x}_{t+s} = k \,|\, \hat{x}_s = i) = p_{ik}(t). \tag{13}$$

Also, making $t \downarrow 0$, we shall show that

$$p(t) \to 1, \tag{14}$$

$$t^{-1}p_{ik}(t) \to 0 \qquad |i - k| > 0, \tag{15}$$

and

$$t^{-1}(1 - p_{ii}(t)) \to +\infty, \tag{16}$$

so that, in the terminology of P. Lévy,[3] *the states i are instantaneous,* $+\infty$ *is fictitious, and no state is stable.*[4]

The counterparts to the Kolmogorov backward and forward differential equations are

$$p_{ij}'(t) = m(r_i)^{-1}\left[\lim_{r_k \downarrow r_i} \frac{p_{kj} - p_{ij}}{r_k - r_i} - \lim_{r_k \uparrow r_i} \frac{p_{ij} - p_{kj}}{r_i - r_k}\right] \tag{17}$$

and

$$p_{ij}'(t) = m(r_j)^{-1}\left[\lim_{r_k \downarrow r_j} \frac{p_{ik} - p_{ij}}{r_k - r_j} - \lim_{r_k \uparrow r_j} \frac{p_{ij} - p_{ik}}{r_j - r_k}\right], \tag{18}$$

which shows how hopeless it is to try to write down backward and forward equations for an arbitrary Markov chain.

2. *Construction.*—Given $v \in C(K)$, write $v \in D(B)$, provided that v satisfies the classical boundary conditions

$$v^+(0) = 0 = v^-(1) \tag{19}$$

and Bv belongs to $C(K)$. This domain $D(B)$ is dense in $C(K)$; the operator B contracted to $D(B)$ generates a semigroup $(T_t;\ t > 0)$ mapping $C(K)$ into $C(K)$; the connection between B and the semigroup is

$$T_t v = v + tBv + c(t) \qquad (t \downarrow 0),\ v \in D(B) \tag{20}$$

(these statements being meant in the customary strong topology); and, setting

$$G_\mu = \int_0^{+\infty} e^{-\mu t}\, T_t\, dt, \qquad \mu > 0, \tag{21}$$

we have the situation described in Section 1. Also, and this will be important below, the Green function $G(\mu, \cdot, \cdot)$ in equation (2) is >0 on $K \times K$ and

$$\int_0^{+\infty} e^{-\mu t}\, P(t, x, M)\, dt = \sum_{r_i \in M} G(\mu, x, r_i) m_i \qquad \mu > 0.^5 \tag{22}$$

Our next step is to prove relation (10). Given $x \in K$, equation (22) makes it clear that $P(t, x, R) = 1$, provided that t is not contained in some exceptional set, N, of measure 0. Choose $t \in N$, and put $M = (t_0;\ P(t_0, x, r_i) < 1, \text{ some } i > 0)$. We have

$$1 > P(t, x, R) \geq \int_R P(t - t_0, x, ds) P(t_0, s, R) = P(t - t_0, x, R) \qquad t_0 \notin M,$$

and, since M has measure 0, this is a contradiction, N is empty, and relation (10) is true.

This brings us to relations (14), (15), and (16). Choose positive $v \in D(B)$, vanishing near $x \in K$ and such that $v = 1$ outside some neighborhood J containing x. Keeping equation (20) in mind, we have

$$P(t, x, K - J) \leq (T_t v)(x) = o(t) \qquad (t \downarrow 0), \tag{23}$$

and this proves relation (15). To prove relation (14), we have merely to show that $p_{ii}(t) \to 1$ $(t \downarrow 0)$. Remembering relations (11) and (15), we have

$$\begin{aligned} o(1) + G(\mu, r_i, r_i) m_i &= \int_0^{+\infty} e^{-\mu t}\, p_{ii}(t + \epsilon)\, dt \\ &= \sum_{k > 0} G(\mu, r_i, r_k) m_k\, p_{ki}(\epsilon) \\ &= o(1) + G(\mu, r_i, r_i)\, m_i\, p_{ii}(\epsilon) \quad (\epsilon \downarrow 0), \end{aligned}$$

and, since $G(\mu, r_i, r_i) > 0$, we see that relation (14)[6] is true.

Coming to relation (16), take $i > 0$ and a positive $v \in D(B)$ such that $v(r_i) = 1$. We have

$$t^{-1}(1 - p_{ii}(t)) \geq t^{-1}(1 - (T_t v)(r_i)) = -(Bv)(r_i) + o(1) \qquad (t \downarrow 0),$$

and, since $-(Bv)(r_i)$ can be taken as large as we please, relation (16) is true.

Remark: According to a theorem of D. Ray,[7] relation (23) implies that the sample functions x_t can be taken continuous with probability 1. Consequently, the sets

$$S_i = (t;\ \hat{x}_t = i) = (t;\ x_t = r_i) \qquad i > 0, \tag{24}$$

are closed and, by a theorem of P. Lévy,[8] nowhere dense.

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[1] W. Feller, "The General Diffusion Operator and Positivity-preserving Semi-groups in One Dimension," *Ann. Math.*, **60**, 417–436, 1954; "The Parabolic Differential Equations and the Associated Semi-groups of Transformations," *ibid.*, **55**, 468–519, 1952; J. L. Doob, *Stochastic Processes* (New York: John Wiley & Sons, 1953).

[2] Given such a measure $m(dx)$, we make the following definition: $Bv = w$ means that the one-sided derivatives v^+ and v^- exist on (0, 1), that $v^+ = v^-$ on $K - R$, and that $v^+(b) - v^-(a) = \Sigma\, w(r_i)m_i$ $(a < b)$, the summation being taken over $(i: r_i \in [a, b])$.

[3] P. Lévy, "Systémes markoviens et stationnaires: Cas dénombrable," *Ann. sci. École norm. supér.*, **68**, 327–381, 1951.

[4] We note that this contradicts P. Lévy's statement (*ibid.*, p. 375) that a chain must have stable states.

[5] Compare W. Feller, "The Parabolic Differential Equations and the Associated Semi-groups of Transformations," *Ann. Math.*, **55**, 468–519, 1952.

[6] This is a special case of a theorem due to J. Doob, "Topics in the Theory of Markov Chains," *Trans. Am. Math. Soc.*, **52**, 37–64, 1942.

[7] D. Ray, "Stationary Markov Processes with Continuous Paths," to appear in *Trans. Am. Math. Soc.* (1956).

[8] P. Lévy, *op. cit.*, p. 373. His argument contains a slip, but it is easy to correct.

ON DIFFERENTIAL GEOMETRY AND HOMOGENEOUS SPACES. II

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We retain the notation of the preceding paper.[1] We will say that $\mathfrak{g}$ is effective relative to $\mathfrak{k}$ if $\mathfrak{k}$ contains no ideal of $\mathfrak{g}$. This is equivalent to π being a faithful representation of $\mathfrak{g}$ or to σ being a faithful representation of $\mathfrak{k}$ on $\mathfrak{p}$ for any decomposition $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$., as for example $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$. Since our concern is with G/K, we will tacitly assume that this is the case.

Now it follows from the proof of Lemma 1 that a necessary and sufficient condition that $\mathfrak{p}$ be natural is that there exists a positive-definite (p.d.) bilinear form B on $\mathfrak{p}$ which satisfies the following: (1) invariance under σu, $u \in K$; (2) $P\ adX\ P$ is skew-symmetric for all $X \in \mathfrak{g}$. Given $\mathfrak{p}$ such that only $\mathfrak{k} \cap \mathfrak{p} = 0$, $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, $[\mathfrak{k}, \mathfrak{p}] \subseteq \mathfrak{p}$, we will call a p.d. bilinear form B on $\mathfrak{p}$ which satisfies only condition 2 *strictly invariant* (s.i.). Even though K may not be connected, the following theorem will imply that both $\mathfrak{p}$ and B are invariant under σu for any $u \in K$, so that $\mathfrak{p}$ is natural; hence the notion of naturalness is a purely Lie algebra concept, and we treat it as such.[2]

Now if B^* is p.d. invariant bilinear form on $\mathfrak{g}$, then if $\mathfrak{p}'$ is the orthocomplement to $\mathfrak{k}$ and B' is the restriction of B^* to $\mathfrak{p}'$, it is clear that B' is s.i. We shall see, however, that $\mathfrak{p}'$ is, in general, a very special natural complement. Nevertheless, one may question whether a s.i. B on a natural $\mathfrak{p}$ arises from an invariant B^* on $\mathfrak{g}$. If we let $\mathfrak{g}(\mathfrak{p}) = \mathfrak{p} + [\mathfrak{p}, \mathfrak{p}]$, then clearly $\mathfrak{g}(\mathfrak{p})$ is an ideal in $\mathfrak{g}$; since one may always find a complementary ideal $\mathfrak{n}$, i.e., $\mathfrak{g} = \mathfrak{n} \oplus \mathfrak{g}(\mathfrak{p})$, and since $\mathfrak{g}(\mathfrak{p}) = \mathfrak{k}_1 + \mathfrak{p}$, where $\mathfrak{k}_1 = \mathfrak{k} \cap \mathfrak{g}(\mathfrak{p})$, it is clear that this question has only to do with $\mathfrak{g}(\mathfrak{p})$.

The following theorem is our main tool in obtaining the results to follow.

THEOREM 4. *Let* $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, $\mathfrak{k} \cap \mathfrak{p} = 0$, $[\mathfrak{k}, \mathfrak{p}] \subseteq \mathfrak{p}$. *Let* B *be strictly invariant on* $\mathfrak{p}$. *Let* $\mathfrak{g}(\mathfrak{p}) = \mathfrak{p} + [\mathfrak{p}, \mathfrak{p}]$, $\mathfrak{k}_1 = \mathfrak{g}(\mathfrak{p}) \cap \mathfrak{k}$, *so that the ideal* $\mathfrak{g}(\mathfrak{p}) = \mathfrak{k}_1 + \mathfrak{p}$.

There exists one and only one invariant bilinear form B^* *on* $\mathfrak{g}(\mathfrak{p})$ *extending* B *and such that* $B^*(\mathfrak{k}_1, \mathfrak{p}) = 0$. *Moreover,* B^* *is nonsingular on* $\mathfrak{g}(\mathfrak{p})$ *and hence on* $\mathfrak{k}_1$.

Proof: Given B, we identify $\wedge^2\mathfrak{p}$ with the Lie algebra of all skew-symmetric operators on $\mathfrak{p}$ by setting $(X \wedge Y)Z = {}^1/_2B(X, Z)Y - {}^1/_2B(Y, Z)X$, for $X, Y, Z \in \mathfrak{p}$. Let B_2 be the p.d. bilinear form on $\wedge^2\mathfrak{p}$ defined by $B_2(A_1, A_2) = -\mathrm{tr}\, A_1A_2$. One verifies that, for $A \in \wedge^2\mathfrak{p}$, $X, Y \in \mathfrak{p}$, $B(A\,X, Y) = B_2(A, X \wedge Y)$.

Now $X \to \sigma X$ may be regarded as an isomorphism of $\mathfrak{k}$ into $\wedge^2\mathfrak{p}$. We define an operator R on $\wedge^2\mathfrak{p}$ by letting $R(X \wedge Y) = \sigma((1 - P)[X, Y])$. We will now show that R is a symmetric operator. Let $X, Y, W, Z \in \mathfrak{p}$. It suffices to show that $B_2(R(X \wedge Y), W \wedge Z) = B_2(R(W \wedge Z), X \wedge Y)$ or, from the above, $B(R(X \wedge Y)W, Z) = B(R(W \wedge Z)X, Y)$. Extend B to $\mathfrak{g}$ for the present by letting $B(\mathfrak{k}, \mathfrak{g}) = B(\mathfrak{g}, \mathfrak{k}) = 0$.

Now, by Jacobi, $B([[X, Y], W], Z) = B([[X, W], Y], Z) + B([X, [Y, W]], Z)$. Since B is s.i. on $\mathfrak{p}$, however, $B([[X, W], Y], Z) = B(Y, [Z, [X, W]])$ and $B([X,[Y, W]], Z) = B(X, [[Y, W], Z])$. But, again by Jacobi, $B([[X, Y], W], Z) = B(Y, [[Z, X], W]) + B(Y, [X, [Z, W]]) + B(X, [[Y, Z], W]) + B(X, [Y, [W, Z]])$. Employing the s.i. of B on $\mathfrak{p}$ and Jacobi again, we get $B([[X, Y], W], Z) = 2B([[W, Z], X], Y) + B([[X, Y], Z], W)$. Hence (*a*) $B([[X, Y], W], Z) = B([[W, Z], X], Y)$. But, using the s.i. of B on $\mathfrak{p}$ again, (*b*) $B([P[X, Y], W], Z) = B(P[X, Y], P[W, Z]) = B([P[W, Z], X], Y)$. Subtracting equation (*b*) from equation (*a*), we obtain $B([(1 - P)[X, Y], W], Z) = B([(1 - P)[W, Z], X], Y)$, or simply $B(R(X \wedge Y)W, Z) = B(R(W \wedge Z)X, Y)$, and hence R is symmetric.

Now for any $X \in \mathfrak{k}$, ad σX is an operator in $\wedge^2\mathfrak{p}$. We assert that R commutes with ad σX for all $X \in \mathfrak{k}$. Indeed, for $X \in \mathfrak{k}$, $W, Z \in \mathfrak{p}$, R ad $\sigma X(W \wedge Z) = R([X, W] \wedge Z + W \wedge [X, Z]) = \sigma((1 - P)([[X, W], Z] + [W, [X, Z]])) = \sigma((1 - P)[X, [W, Z]]) = [\sigma X, R(W \wedge Z)] = \mathrm{ad}\ \sigma X\, R(W \wedge Z)$. Hence R commutes with ad σX for all $X \in \mathfrak{k}$.

Now $\mathfrak{k}_1 = \mathfrak{g}(\mathfrak{p}) \cap \mathfrak{k}$. It follows immediately that the range of R is exactly $\sigma\mathfrak{k}_1$. But, since R is symmetric, R must be nonsingular on $\sigma\mathfrak{k}_1$. Let R' equal R^{-1} on $\sigma\mathfrak{k}_1$, and let it equal zero on the orthocomplement of $\sigma\mathfrak{k}_1$. Clearly R', too, must commute with all ad σX, $X \in \mathfrak{k}$, and $RR' = R'R$ is the orthogonal projection $\vee^2\mathfrak{p} \to \sigma\mathfrak{k}_1$.

We now define B^* by letting (1) $B^* = B$ on $\mathfrak{p}$, (2) $B^*(\mathfrak{k}_1, \mathfrak{p}) = B^*(\mathfrak{p}, \mathfrak{k}_1) = 0$, and (3) $B^*(X, Y) = B_2(R'\,\sigma X, \sigma Y)$ for $X, Y \in \mathfrak{k}_1$.

In checking invariance, the only nonobvious identity is $B^*([X, Y], Z) = B^*(X, [Y, Z])$ when $X \in \mathfrak{k}_1$, $Y, Z \in \mathfrak{p}$. But then $B^*([X, Y], Z) = B([X, Y], Z) = B_2(\sigma X, Y \wedge Z) = B_2(RR'\,\sigma X, Y \wedge Z) = B_2(R'\,\sigma X, R\,Y \wedge Z) = B_2(R'\,\sigma X, \sigma(1 - P)[Y, Z]) = B^*(X, (1 - P)[Y, Z]) = B^*(X,[Y, Z])$.

Now assume that B^{**} likewise satisfies the conditions of Theorem 1. Let $\hat{B} = B^{**} - B^*$. Then $\hat{B}(\mathfrak{g}(\mathfrak{p}), \mathfrak{p}) = 0$, and also $\hat{B}([\mathfrak{p}, \mathfrak{p}], \mathfrak{g}(\mathfrak{p})) = \hat{B}(\mathfrak{p}, [\mathfrak{p}, \mathfrak{g}(\mathfrak{p})]) = 0$. Thus $\hat{B} = 0$, and hence B^* is unique.

Finally, if $X \in \mathfrak{k}_1$, since $\mathfrak{g}$ is effective relative to $\mathfrak{k}$, there exists $Y \in \mathfrak{p}$ such that $[X, Y] \neq 0$. But then $B^*(X, [Y, [X, Y]]) = B([X, Y], [X, Y]) \neq 0$, and hence B^* is nonsingular. *Q.E.D.*

Note: Even though B is p.d. on $\mathfrak{p}$ and B^* is nonsingular on $\mathfrak{g}(\mathfrak{p})$, it is not always true that B^* is p.d.

Now, given a natural decomposition $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, we will call $\mathfrak{p}$ *pervasive in* $\mathfrak{g}$ if $\mathfrak{g}(\mathfrak{p}) = \mathfrak{g}$. Observe that, relative to $\mathfrak{k}_1$, $\mathfrak{g}(\mathfrak{p})$ is effective, $\mathfrak{g}(\mathfrak{p}) = \mathfrak{k}_1 + \mathfrak{p}$ is a natural decomposition, and of course $\mathfrak{p}$ is pervasive in $\mathfrak{g}(\mathfrak{p})$. The following theorem reduces the study of natural complements to those which are pervasive.

Let $\mathfrak{l} = \{X \in \mathfrak{p} | [\mathfrak{k}_1, X] = 0\}$. Let B^* be as in Theorem 4. Then $B^*(\mathfrak{k}_1, [\mathfrak{l}, \mathfrak{p}]) = B^*([\mathfrak{k}_1, \mathfrak{l}], \mathfrak{p}) = 0$ implies that $[\mathfrak{l}, \mathfrak{p}] \subseteq \mathfrak{p}$, and hence $\mathfrak{l}$ is a subalgebra of $\mathfrak{g}$. Let $u = [\mathfrak{g}(\mathfrak{p}), \mathfrak{g}(\mathfrak{p})]$. The subalgebra $\mathfrak{l} \cap u \subseteq \mathfrak{g}(\mathfrak{p})$ plays for us a central role.

COROLLARY 4. *Let* $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ *be natural. Let* $\mathfrak{n}$ *be the set of elements* $X \in \mathfrak{g}$ *such that* (1) X *commutes with* $\mathfrak{g}(\mathfrak{p})$ *and* (2) $PX \in \mathfrak{l} \cap u$. *Then* $\mathfrak{n}$ *is an ideal in* $\mathfrak{g}$ *complementary to* $\mathfrak{g}(\mathfrak{p})$, *so that* $\mathfrak{g} = \mathfrak{n} \oplus \mathfrak{g}(\mathfrak{p})$.

Furthermore, the mapping $X \to -PX$ *is an isomorphism of* $\mathfrak{n}$ *into* $\mathfrak{l} \cap u$. *Moreover, the mapping* $X \to (1 - P)X$ *is an isomorphism of* $\mathfrak{n}$ *into* $\mathfrak{k}$, *and if* $\mathfrak{k}_0 = (1 - P)\mathfrak{n}$, *then* $\mathfrak{k} = \mathfrak{k}_0 \oplus \mathfrak{k}_1$.

Note: If, conversely, given $\mathfrak{g}_1 = \mathfrak{k}_1 + \mathfrak{p}$ ($\mathfrak{p}$ natural and pervasive) and also given a Lie algebra $\mathfrak{n}$ and an isomorphism $\tau: \mathfrak{n} \to \mathfrak{l} \cap u \subseteq \mathfrak{g}_1$, then, defining $\mathfrak{k}_0 \subseteq \mathfrak{n} \oplus \mathfrak{g}_1 = \mathfrak{g}$ by $\mathfrak{k}_0 = \{X + \tau X | X \in \mathfrak{n}\}$ and letting $\mathfrak{k} = \mathfrak{k}_0 + \mathfrak{k}_1$ ($= \mathfrak{k}_0 \oplus \mathfrak{k}_1$), one has the result that $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ is a nonpervasive natural decomposition, and, of course, by Corollary 4 the most general one can be so constructed.

Among all the natural complements, one may now characterize those which are pervasive by

COROLLARY 5. *Let* $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, $\mathfrak{k} \cap \mathfrak{p} = 0$, $[\mathfrak{k}, \mathfrak{p}] \subseteq \mathfrak{p}$. *Then* $\mathfrak{p}$ *is natural and pervasive if and only if there exists a nonsingular invariant bilinear form* B^* *on* $\mathfrak{g}$ *such that* (1) B^* *is p.d. on* $\mathfrak{p}$ *and* (2) $B^*(\mathfrak{k}, \mathfrak{p}) = 0$.

As a second corollary, we generalize a well-known theorem on compact semi-simple group spaces with the (0) connection.

COROLLARY 6. *Let* $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, *where* $\mathfrak{p}$ *is natural and pervasive. Let* $M = G/K$ *be given the corresponding canonical affine connection (it is Riemannian, and we may assume that* M *is given a metric inducing it). Let* $\mathfrak{g}^*$ *be the Lie algebra of all infinitesimal motions on* M.

Then the centralizer $\mathfrak{c}^*$ *of* $\pi(\mathfrak{g})$ *in* $\mathfrak{g}^*$ *may be written* $\mathfrak{c}^* = \pi(\mathfrak{c}) \oplus \mathfrak{n}$, *where* (1) $\mathfrak{c}$ *is the center of* $\mathfrak{g}$, (2) $\mathfrak{n} \cap \pi(\mathfrak{g}) = 0$, *and* (3) $\mathfrak{n}$ *is isomorphic to* $\mathfrak{l} \cap u \subseteq \mathfrak{p}$ ($u = [\mathfrak{g}, \mathfrak{g}]$, $\mathfrak{l} = \{X \in \mathfrak{p} | [\mathfrak{k}, X] = 0\}$) *under the mapping* $\tau(X^*) = -X^*|_o \in \mathfrak{p}$ *for* $X^* \in \mathfrak{n}$.

Reducibility: Let $M = G/K$ be as in Corollary 6 above. Let $\mathfrak{p}_1 \subseteq \mathfrak{p}$ be a subspace invariant under $\mathfrak{f}_o$ (the holonomy algebra). Let B^* be as in Corollary 5, and let $\mathfrak{p}_2$ be the orthocomplement to $\mathfrak{p}_1$. Thus $\mathfrak{p} = \mathfrak{p}_1 + \mathfrak{p}_2$, and $\mathfrak{p}_2$ is invariant under $\mathfrak{f}_o$. Now by Theorem 1, $P[X_1, X_2] \in \mathfrak{p}_1 \cap \mathfrak{p}_2 = 0$ if $X_1 \in \mathfrak{p}_1$, $X_2 \in \mathfrak{p}_2$. Thus $[X_1, X_2] \in \mathfrak{k}$. However, if $Y \in \mathfrak{k}$, then $B^*(Y, [X_1, X_2]) = B([Y, X_1], X_2) = 0$. But since B^* is nonsingular on $\mathfrak{k}$, then $[X_1, X_2] = 0$. Thus $[\mathfrak{p}_1, \mathfrak{p}_2] = 0$. It follows that if $\mathfrak{g}_1 = \mathfrak{p}_1 + [\mathfrak{p}_1, \mathfrak{p}_1]$ and $\mathfrak{g}_2 = \mathfrak{p}_2 + [\mathfrak{p}_2, \mathfrak{p}_2]$, then $\mathfrak{g}_1$ and $\mathfrak{g}_2$ are ideals in $\mathfrak{g}$. Hence $\mathfrak{p}_1$ and $\mathfrak{p}_2$ are invariant under ψ_o, by Theorem 2. Also, since B^* is nonsingular on $\mathfrak{g}_1$ and $\mathfrak{g}_2$ and $B^*(\mathfrak{g}_1, \mathfrak{g}_2) = 0$, we obtain the following generalization of a theorem of Cartan on symmetric spaces.

THEOREM 5. *Let* $M = G/K$ *be as in Corollary* 6. *Let* ψ_o *be the holonomy group at* o. *A subspace* $\mathfrak{p}_1 \subseteq \mathfrak{p}$ *is invariant under* ψ_o *if and only if it is of the form* $\mathfrak{p}_1 = P\mathfrak{g}_1$, *where* $\mathfrak{g}_1$ *is an ideal of* $\mathfrak{g}_1$ *such that* $\mathfrak{g}_1 = (1 - P)\mathfrak{g}_1 + P\mathfrak{g}_1$.

Moreover, if $\sum_{i=1}^{m} \mathfrak{p}_i = \mathfrak{p}$ *is a direct sum decomposition of* $\mathfrak{p}$ *into invariant subspaces of* ψ_o *(irreducible or not), then* (1) $\mathfrak{g}_i = \mathfrak{p}_i + [\mathfrak{p}_i, \mathfrak{p}_i]$, $i = 1, \ldots, m$, *are ideals in* $\mathfrak{g}$; (2) $\mathfrak{g}_i = \mathfrak{k}_i + \mathfrak{p}_i$ *is a natural (pervasive) decomposition of* $\mathfrak{g}_i$; *and* (3) $\mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \ldots \oplus \mathfrak{g}_m$ *and* $\mathfrak{k} = \mathfrak{k}_1 \oplus \mathfrak{k}_2 \oplus \ldots \oplus \mathfrak{k}_m$ *where* $\mathfrak{k}_i = \mathfrak{g}_i \cap \mathfrak{k}$.

Finally, if $\mathfrak{c}$ *is the center of* $\mathfrak{g}$, *then* $\mathfrak{p} \cap \mathfrak{c}$ *is the space of fixed vectors for* ψ_o.

The de Rham decomposition theorem becomes

COROLLARY 7. *Let* $M = G/K$ *be as in Corollary* 6. *Let* $\mathfrak{g}_i$, $i = 1, \ldots, m$, *be as in Theorem* 5, *except that the* $\mathfrak{p}_i$*'s are assumed irreducible and* G *is assumed simply connected. Then, if* $G_i \subseteq G$ *corresponds to* $\mathfrak{g}_i \subseteq \mathfrak{g}$, G_i *is a compact normal subgroup of* G. *Moreover, if* $K_i = K \cap G_i$, *and* $G_i/K_i = M_i$ *is given the canonical affine connection with respect to* $\mathfrak{g}_i = \mathfrak{k}_i + \mathfrak{p}_i$, *then the de Rham decomposition may be realized by* $M = M_1 \times M_2 \times \ldots \times M_n$.

Recalling that a manifold with a Riemannian connection is irreducible if and only if a metric tensor inducing the connection is uniquely determined up to a scalar multiple, we have

COROLLARY 8. *Let* $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, $\mathfrak{p}$ *natural and pervasive. Then, up to scalar multiple, there exists exactly one s.i.* B *on* $\mathfrak{p}$ *if and only if there exists no nontrivial ideal* $\mathfrak{g}' \subseteq \mathfrak{g}$ *such that* $P\mathfrak{g}' \subseteq \mathfrak{g}'$.

Finally, by virtue of Theorems 4 and 5, we have

COROLLARY 9. *Assume that* $\mathfrak{g}$ *is simple and that* K *is an arbitrary compact subgroup of* G. *Then* (1) $\mathfrak{g}$ *is effective relative to* $\mathfrak{k}$, (2) *there exists exactly one natural (necessarily pervasive) complement* $\mathfrak{p}$, *and* (3) *the unique natural affine connection on* G/K *is irreducible.*

Conversely, if G *contains a subgroup* K *such that statements* 1, 2, *and* 3 *hold, then* $\mathfrak{g}$ *is simple.*

[1] B. Kostant, "On Differential Geometry and Homogeneous Spaces. I," these PROCEEDINGS, **42**, 258, 1956.

[2] All bilinear forms in this note are assumed to be symmetric.

THE DEGREE OF AN IRREDUCIBLE REPRESENTATION OF S_n

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1. As is well known, the irreducible representations of S_n are in one-to-one correspondence with the Young *diagrams* $[\lambda]$, where $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_n$ and $\lambda_1 + \lambda_2 + \ldots + \lambda_n = n$. If we arrange the numbers $1, 2, \ldots, n$ in $[\lambda]$, we have a *tableau* which is *standard* if the integer in the (i, j)-position is less than that in the (k, l)-position for $i \leqslant k$ and $j \leqslant l$. The actual matrices of $[\lambda]$ can be written in terms of these standard tableaux t_λ according to Young, and the degree f_λ of $[\lambda]$ is found by enumerating these t_λ. Young's proof of the formula

$$f_\lambda = n! \frac{\prod_{i<j} (h_{i1} - h_{j1})}{\prod h_{i1}!} \tag{1.1}$$

involves an unpleasant induction. Here $h_{ij} = (\lambda_i - i) + (\lambda_j' - j) + 1$, where λ_j' is the length of the jth column of $[\lambda]$; thus h_{ij} is the length of the *hook* consisting of those nodes to the right of, below, and including the (i, j)-position. Frobenius also obtained equation (1.1) from a different point of view.

If we write the alternating function in the numerator of expression (1.1) as a determinant, we are led to Feit's version:[1]

$$f = n!|z_{ij}|, \tag{1.2}$$

where $z_{ij} = 1/(\lambda_j - j + i)!$ and $z_{ij} = 1$ if $\lambda_j - j + i = 0$, $z_{ij} = 0$ if $\lambda_j - j + i < 0$. Feit's proof was again by induction. It is the purpose of this note to prove relation (1.2), and hence relation (1.1), directly.

2. If we denote the permutation representation of S_n induced by the identity representation of the subgroup $S_{\lambda_1} \times S_{\lambda_2} \times \ldots \times S_{\lambda_n}$ by $[\lambda_1]\cdot[\lambda_2]\cdot\ldots\cdot[\lambda_n]$, then we have the reduction

$$[\lambda_1]\cdot[\lambda_2]\cdot\ldots\cdot[\lambda_n] = \Pi(1 - R_{ij})^{-1}[\lambda], \tag{2.1}$$

where R_{ij} raises a symbol from the jth row of $[\lambda]$ to the ith row. Such a term is taken to be zero (i) if any row becomes longer than a preceding row or (ii) if two symbols from the same row appear in the same column. We consider $[\lambda_1]\cdot[\lambda_2]\cdot\ldots\cdot[\lambda_n]$ as a *skew* diagram in which the $[\lambda_i]$ are disjoint, yielding

$$\frac{n!}{\lambda_1!\,\lambda_2!\ldots\lambda_n!} \tag{2.2}$$

standard skew tableaux, in terms of which we obtain the usual permutation matrices for the transpositions $(r, r+1)$, by setting Young's $\rho = 0$ if r and $r+1$ lie in disjoint constituents.

In a recent note[2] it was observed that equation (2.1) can be inverted:

$$[\lambda] = \Pi(1 - R_{ij})[\lambda_1]\cdot[\lambda_2]\cdot\ldots\cdot[\lambda_n], \tag{2.3}$$

where now restrictions i and ii do not apply. For example, if $r + s + t = n$,

$$\begin{aligned}[r, s, t] &= (1 - R_{12})(1 - R_{13})(1 - R_{23})[r]\cdot[s]\cdot[t] \\ &= (1 - R_{12} - R_{13} - R_{23} + R_{12}R_{13} + R_{13}R_{23} + R_{12}R_{23} - R_{12}R_{13}R_{23}) \\ &\qquad [r]\cdot[s]\cdot[t] \\ &= [r]\cdot[s]\cdot[t] - [r+1]\cdot[s-1]\cdot[t] - [r+1]\cdot[s]\cdot[t-1] - [r]\cdot[s+1]\cdot \\ &\quad [t-1] + [r+2]\cdot[s-1]\cdot[t-1] + [r+1]\cdot[s+1]\cdot[t-2] + \\ &\quad [r+1]\cdot[s]\cdot[t-1] - [r+2]\cdot[s]\cdot[t-2],\end{aligned}$$

and substituting the degrees from expression (2.2), we obtain precisely expression (1.2), noting that the terms arising from the operators R_{13} and $R_{12}R_{23}$ cancel.

In the general case we can distinguish $n!$ *determinantal* terms in the expansion of expression (2.3). The corresponding operators are uniquely defined, and the degrees given by expression (2.2) are precisely the terms appearing in relation (1.2) with correct sign. The remaining terms are *nondeterminantal*, and the corresponding operators are subject to expansion and contraction as in the case of R_{13} and $R_{12}R_{23}$. All such terms cancel. The details of this proof of relation (1.2) will be given elsewhere.

3. In a recent paper[3] it was shown that the factors in the numerator of expression (1.1) do *not* represent hooks in $[\lambda]$, and, dividing them out, we have

$$f_\lambda = \frac{n!}{H} \tag{3.1}$$

where H is the product of the lengths of *all* the hooks in $[\lambda]$. This formula is of general interest, since it gives a simple interpretation for the quotient $n!/f_\lambda$. Note that expression (2.2) is the special case of expression (3.1) appropriate to the skew diagram $[\lambda_1] \cdot [\lambda_2] \cdot \ldots \cdot [\lambda_n]$.

It should be remarked in conclusion that an operator approach to the reduction of the skew diagram $[\lambda] - [\mu]$ is also available. Indeed, Feit's formula was devised to yield

$$f_{\lambda-\mu} = n! \left| z_{ij} \right|, \tag{3.2}$$

where $z_{ij} = 1/(\lambda_j - j - \mu_i + i)!$, with the same conditions as before. Setting $\rho = 0$ when r and $r+1$ appear in disjoint constituents of a standard skew diagram yields the matrices of the induced representation corresponding to any $[\lambda] - [\mu]$. It is interesting to compare these ideas with the corresponding theory of Schur functions $\{\lambda\}$ as developed by D. E. Littlewood.[4] One might add that this operator approach does yield the enumeration of standard tableaux without the intervention of any other machinery. Moreover, the relation (2.3) provides an immediate proof that the character of a cycle of length n in $[n-r, 1^r]$ is $(-1)^r$; thus the motion of a *hook* derives its significance directly from the raising operator which has proved so important in the modular theory.[5]

[1] W. Feit, *Proc. Am. Math. Soc.*, **4**, 740–744, 1953; F. D. Murnaghan, *The Theory of Group Representations* (Baltimore, 1938), Chaps. V, VII.

[2] G. de B. Robinson and O. E. Taulbee, these PROCEEDINGS, **40**, 723–726, 1954.

[3] J. S. Frame, G. de B. Robinson, and R. M. Thrall, *Can. J. Math.*, **6**, 316–324, 1954.

[4] D. E. Littlewood, *The Theory of Group Characters* (Oxford, 1940), chap. vi.

[5] G. de B. Robinson and O. E. Taulbee, these PROCEEDINGS **41**, 596–598, 1955.

ON A GENERALIZATION OF THE NOTION OF MANIFOLD

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In the following, I shall introduce a notion of V-manifold, which is a generalization of the notion of manifold as well as of that of quotient space of a manifold with respect to a properly discontinuous group of transformations. I shall also indicate how de Rham's theorem and Poincaré's duality theorem can be generalized to the case of V-manifolds.

1. Let $\mathfrak{B}$ be a Hausdorff space. A *local uniformizing system* (l.u.s.) $\{\tilde{U}, G, \varphi\}$ for an open set $U \subset \mathfrak{B}$ is by definition a collection of the following objects:

$\tilde{U}$: a connected open subset of R^n.

G: a finite group of linear transformations of $\tilde{U}$ onto itself. We assume that the set of all fixed points of G is of dimension $\leqq n-2$.

φ: a continuous map $\tilde{U} \to U$ such that $\varphi \circ \sigma = \varphi$ for all $\sigma \in G$. Then φ induces a map from the quotient space $G\backslash\tilde{U}$ onto U, which we assume to be a homeomorphism.

Let $\{\tilde{U}, G, \varphi\}$, $\{\tilde{U}', G', \varphi'\}$ be l.u.s. for U, U', respectively, and let $U \subset U'$. By an *injection* λ: $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}', G', \varphi'\}$ we mean a C^∞-isomorphism λ from $\tilde{U}$ onto a subdomain of $\tilde{U}'$ such that for any $\sigma \in G$ there exists $\sigma' \in G'$ satisfying the relation $\lambda \circ \sigma = \sigma' \circ \lambda$ and such that $\varphi = \varphi' \circ \lambda$. Then σ' is uniquely determined by σ, and the correspondence $\sigma \to \sigma'$ defines an isomorphism from G into G'. If $U = U'$, then the C^∞-isomorphism λ: $\tilde{U} \to \tilde{U}'$ and the associated isomorphism $G \to G'$ become onto, and λ^{-1} is also an injection $\{\tilde{U}', G', \varphi'\} \to \{\tilde{U}, G, \varphi\}$. In this case $\{\tilde{U}, G, \varphi\}$, $\{\tilde{U}', G', \varphi'\}$ are said to be *equivalent*.

By the definition above we can prove the following:

LEMMA 1. *Let λ, μ be two injections $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}', G', \varphi'\}$. Then there exists a uniquely determined $\sigma' \in G'$ such that $\mu = \sigma' \circ \lambda$.*

If λ: $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}', G', \varphi'\}$, λ': $\{\tilde{U}', G', \varphi'\} \to \{\tilde{U}'', G'', \varphi''\}$ are injections, then $\lambda' \circ \lambda$ becomes an injection $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}'', G'', \varphi''\}$.

2. DEFINITION. *A C^∞ V-manifold is a composite concept formed of a connected Hausdorff space $\mathfrak{V}$ and a family $\mathfrak{F}$ of l.u.s. for open sets in $\mathfrak{V}$ satisfying the following conditions:*

(1) *Let $\{\tilde{U}, G, \varphi\}$, $\{\tilde{U}', G', \varphi'\} \in \mathfrak{F}$ be l.u.s. for U, U', respectively, and let $U \subset U'$. Then there exists an injection λ: $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}', G', \varphi'\}$.*

(2) *The $\mathfrak{F}$-uniformized open sets, i.e., the open sets U for which there exist l.u.s. $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$, form a basis of open sets in $\mathfrak{V}$.*

Two families $\mathfrak{F}$, $\mathfrak{F}'$ of l.u.s. satisfying conditions 1 and 2 are said to be *equivalent* if $\mathfrak{F} \cup \mathfrak{F}'$ satisfies condition 1. Equivalent families are regarded as defining one and the same V-manifold structure on $\mathfrak{V}$.

In a similar way, we can also define real or complex analytic V-manifolds. A V-manifold is called *orientable* if we can assign an orientation of $\tilde{U}$ for each $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$ such that every possible injection λ in condition 1 preserves these orientations. Thus a complex analytic V-manifold is always orientable.

An ordinary manifold is a special case of V-manifold where every group G in $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$ reduces to the unity group. Moreover, it can easily be proved that if $\tilde{\mathfrak{V}}$ is an analytic manifold and $\mathfrak{G}$ is a properly discontinuous group of analytic automorphisms of $\tilde{\mathfrak{V}}$, then the quotient space $\mathfrak{G}\backslash\tilde{\mathfrak{V}}$ possesses canonically an analytic V-manifold structure.

3. Let $\mathfrak{V}$ be a C^∞ V-manifold with a defining family $\mathfrak{F}$ of l.u.s.. For $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$, we denote by $D_{\tilde{U}}^p$ the module of all G-invariant C^∞-forms of degree p on $\tilde{U}$. Let $\{\tilde{U}, G, \varphi\}$, $\{\tilde{U}', G', \varphi'\} \in \mathfrak{F}$ be l.u.s. for U, U', respectively, $U \subset U'$, and let λ be an injection $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}', G', \varphi'\}$. Then it is quite clear that for any $\tilde{\omega} \in D_{\tilde{U}'}^p$, we have $\tilde{\omega} \circ \lambda \in D_{\tilde{U}}^p$. Since $\tilde{\omega} \circ \sigma' = \tilde{\omega}$ for $\sigma' \in G'$ and λ is determined up to $\sigma' \in G'$ by Lemma 1, the correspondence $\tilde{\omega} \to \tilde{\omega} \circ \lambda$ is independent of the choice of λ. Denoting this homomorphism $D_{\tilde{U}'}^p \to D_{\tilde{U}}^p$ by $\lambda^*_{\tilde{U}\tilde{U}'}$, we have $\lambda^*_{\tilde{U}\tilde{U}} = 1$, and $\lambda^*_{\tilde{U}\tilde{U}'} \circ \lambda^*_{\tilde{U}'\tilde{U}''} = \lambda^*_{\tilde{U}\tilde{U}''}$ for $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}', G', \varphi'\} \to \{\tilde{U}'', G'', \varphi''\}$. In particular, if $U = U'$, then $\lambda^*_{\tilde{U}\tilde{U}'}$: $D_{\tilde{U}'}^p \to D_{\tilde{U}}^p$ is an isomorphism onto.

Hence the system $\{D_{\tilde{U}}^p, \lambda^*_{\tilde{U}\tilde{U}'}\}$ ($\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$) defines a sheaf D^p on $\mathfrak{V}$ which we call the sheaf of germs of C^∞-forms of degree p on $\mathfrak{V}$. For any open set $U \subset \mathfrak{V}$ an element ω of the module D_U^p of the sections of D^p on U is called a *C^∞-form* of degree p on U. A C^∞-form f of degree 0 on U can be regarded as a (real-valued) function on U, which we call a *C^∞-function*. If U is uniformized by $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$, then D_U^p is isomorphic to $D_{\tilde{U}}^p$ in a canonical way. We can define the operations d, $\wedge$ on the graded module $D_U = \sum_p D_U^p$ in a natural manner.

4. Let $\mathfrak{W}$ be another C^∞ V-manifold defined by a family $\mathfrak{K}$ of l.u.s.

DEFINITION. *A map f from $\mathfrak{V}$ into $\mathfrak{W}$ is called a C^∞-map if the following conditions are satisfied: Let $x \in \mathfrak{V}$, and let V be a neighborhood of $f(x)$ which is uniformized by $\{\tilde{V}, H, \psi\} \in \mathfrak{K}$; then there exists a neighborhood U of x uniformized by $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$ and a C^∞-map $\tilde{f}$ from $\tilde{U}$ into $\tilde{V}$ such that $f \circ \varphi = \psi \circ \tilde{f}$.*

Clearly this definition does not depend on the choice of the families $\mathfrak{F}, \mathfrak{K}$. Moreover we can prove the following lemma.

LEMMA 2. *The notations being as above, let $\tilde{\omega} \in D_{\tilde{V}}^p$. Then $\tilde{\omega} \circ \tilde{f}$ is independent of the choice of $\tilde{f}$ satisfying the above condition.*

It follows, in particular, that $\tilde{\omega} \circ \tilde{f} \in D_{\tilde{U}}^p$. Furthermore, suppose that $\{\tilde{U}', G', \varphi'\} \in \mathfrak{F}$, $\{\tilde{V}', H', \psi'\} \in \mathfrak{K}$, λ: $\{\tilde{U}, G, \varphi\} \to \{\tilde{U}', G', \varphi'\}$, μ: $\{\tilde{V}, H, \psi\} \to \{\tilde{V}', H', \psi'\}$ and that there exists a C^∞-map $\tilde{f}'$: $\tilde{U}' \to \tilde{V}'$ such that $f \circ \varphi' = \psi' \circ \tilde{f}'$. Then, for $\tilde{\omega} \in D_{\tilde{V}'}^p$, we have $\tilde{\omega} \circ \tilde{f}' \circ \lambda = \tilde{\omega} \circ \mu \circ \tilde{f}$.

Hence, if $U \subset \mathfrak{V}$, $V \subset \mathfrak{W}$ are any open sets such that $f(U) \subset V$ and $\omega \in D_V^p$ we can define $\omega \circ f \in D_U^p$. It is clear that $(d\omega) \circ f = d(\omega \circ f)$, $(\omega \wedge \eta) \circ f = (\omega \circ f) \wedge (\eta \circ f)$.

5. A C^∞ singular simplex $s = [f; a_0, \ldots, a_p]$ of dimension p in $\mathfrak{V}$ is defined as usual by a C^∞-map f from a neighborhood of a Euclidean simplex $[a_0, \ldots, a_p]$ into $\mathfrak{V}$. Then we define the integral $\int_s \omega$ of a C^∞-form ω of degree p on s by the formula

$$\int_s \omega = \int_{[a_0, \ldots, a_p]} \omega \circ f.$$

If the carrier of s is contained in U uniformized by $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$ and $\tilde{f}$ is a C^∞-map from a neighborhood of $[a_0, \ldots, a_p]$ into $\tilde{U}$ such that $f = \varphi \circ \tilde{f}$, then

$$\int_s \omega = \int_{\tilde{s}} \tilde{\omega},$$

where $\tilde{s} = [\tilde{f}; a_0, \ldots, a_p]$ is a C^∞ singular simplex in $\tilde{U}$ and $\tilde{\omega} = \omega \circ \varphi \in D_{\tilde{U}}^p$. We can prove Stoke's formula.

We denote by $S = \sum_p S_p$ the graded module (with boundary operator b) of locally finite C^∞ singular chains in $\mathfrak{V}$.

6. Now let $\mathfrak{V}$ be a paracompact C^∞ V-manifold. Then, for any (open) covering $\mathfrak{U} = \{U_i\}_{i \in I}$ of $\mathfrak{V}$, we can construct a locally finite C^∞-partition of unity $\{f_i\}_{i \in I}$ such that the support of f_i is contained in U_i.

We call a C^∞-map Φ: $U \times R \to U$ a C^∞-retraction if $\Phi(x, t) = x$ for $t \geqq 1$, and x_0 for $t \leqq 0$. If $\{\tilde{U}, G, \varphi\} \in \mathfrak{F}$ is such that $\tilde{U}$ is a Euclidean ball, then $\tilde{U}$ has a C^∞-retraction $\tilde{\Phi}$ such that $\tilde{\Phi}(\sigma(\tilde{x}), t) = \sigma(\tilde{\Phi}(\tilde{x}, t))$ for $\sigma \in G$, and so $U = \varphi(\tilde{U})$ has a C^∞-retraction Φ defined by $\Phi \circ (\varphi \times 1) = \varphi \circ \tilde{\Phi}$. Hence open sets in $\mathfrak{V}$ having a C^∞-retraction form a basis of open sets in $\mathfrak{V}$. We can prove easily

LEMMA 3. *For any covering* $\mathfrak{U} = \{U_i\}_{i \in I}$ *of* $\mathfrak{V}$, *there exists a covering* $\mathfrak{U}' = \{U_j'\}_{j \in J}$ *such that*

(1) *Each* U_j' *has a* C^∞*-retraction.*

(2) *There exists a map* $\tau: J \to I$ *such that if* $U_{j_0}' \cap \ldots \cap U_{j_p}' \neq \emptyset$, *then* $U_{j_0}' \subset U_{\tau j_0} \cap \ldots \cap U_{\tau j_p}$.

7. Let $H^p(\mathfrak{U}, R)$ and $H_p(\mathfrak{U}, R)$ be the Čech cohomology and homology group, respectively, of a covering $\mathfrak{U}$ of $\mathfrak{V}$ with coefficients in R. We can prove by the method of Weil[1] that for any covering $\mathfrak{U}$ there exists a canonical homomorphism

$$H^p(\mathfrak{U}, R) \to H^p(D_{\mathfrak{V}}),$$

which is an isomorphism onto if $\mathfrak{U}$ is a simple covering, i.e., if every $U_{i_0} \cap \ldots \cap U_{i_p}$, $U_{i_\nu} \in \mathfrak{U}$, has a C^∞-retraction.

Let $H^p(\mathfrak{V}, R)$ be the inductive limit group of $H^p(\mathfrak{U}, R)$. Then, by Lemma 3, we can prove

THEOREM 1. $H^p(D_{\mathfrak{V}})$ *is isomorphic to* $H^p(\mathfrak{V}, R)$ *canonically.*

Next, let $H_p(\mathfrak{V}, R)$ be the projective limit group of $H_p(\mathfrak{U}, R)$; then we have, similarly,

THEOREM 2. $H_p(S)$ *is isomorphic to* $H_p(\mathfrak{V}, R)$ *canonically.*

We have quite analogous results, restricting cochains and chains to finite ones in the construction of $H^p(\mathfrak{V}, R)$ and $H_p(\mathfrak{V}, R)$, respectively, and also restricting C^∞-forms and C^∞-chains to those with compact supports in defining $H^p(D_{\mathfrak{V}})$ and $H_p(S)$, respectively.

Since $H^p(\mathfrak{V}, R)$ and $H_p(\mathfrak{V}, R)$, one of which is constructed in the restricted sense, are mutually dual, the same relation holds between $H^p(D_{\mathfrak{V}})$ and $H_p(S)$. Writing this duality explicitly, we can see that the inner product is given by the integral $\int_t \omega$, ω being a closed C^∞-form of degree p and t a C^∞-cycle of dimension p. We have thus the first and second theorems of de Rham.

8. Let $\mathfrak{V}$ be orientable. Then, for $\omega \in D_{\mathfrak{V}}^n$ with a compact carrier, we can define the integral $\int_{\mathfrak{V}} \omega$ as follows: if the carrier of ω is contained in U uniformized by $\{\bar{U}, G, \varphi\} \in \mathfrak{F}$, we put

$$\int_{\mathfrak{V}} \omega = \frac{1}{N_G} \int_{\bar{U}} \bar{\omega},$$

where $\bar{\omega} = \omega \circ \varphi$ and N_G is the order of G. Let $\{f_i\}_{i \in I}$ be a locally finite C^∞-partition of unity such that the support of f_i is contained in an $\mathfrak{F}$-uniformized open set U_i. For general ω, we put

$$\int_{\mathfrak{V}} \omega = \sum_{i \in I} \int_{\mathfrak{V}} f_i \omega.$$

Then we can prove easily that this definition does not depend on the choice of $\{f_i\}$.

From what has been proved above, it follows that $\mathfrak{V}$ is a "homological manifold" for real coefficients. This means that every $x \in \mathfrak{V}$ has arbitrarily small open neighborhoods, each one of which has the same cohomology with compact carriers (over real coefficients) as the space R^n. Then, assuming for simplicity that V is compact, we can prove (again by Weil's method) the following theorem:

THEOREM 3. *$H^{n-p}(D_{\mathfrak{B}})$ is isomorphic to $H_p(\mathfrak{B}, R)$ canonically.*

Hence $H^p(D_{\mathfrak{B}})$ and $H^{n-p}(D_{\mathfrak{B}})$ are mutually dual. We can see that the inner product is given by the integral $\pm\int_{\mathfrak{B}} \omega \wedge \eta$, ω and η being closed C^∞-forms of degree p and $n - p$, respectively. We have thus the duality theorem of Poincaré on Betti groups.

[1] A. Weil, "Sur les théorèmes de de Rham," *Comm. Math. Helv.*, **26**, 119–145, 1952.

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*THE KINETIC ENERGY OF RELATIVE MOTION**

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The kinetic energy for a system of N particles is

$$T = \frac{1}{2}\sum_i m_i \dot{r}_i^2, \tag{1}$$

where m_i is the mass of the ith particle and $\vec{r}_i$ is its position. Very often it is desirable to separate off the kinetic energy of the center of mass and express the relative kinetic energy in terms of a set of relative co-ordinates. There are many different ways of defining the relative co-ordinates, each having characteristic advantages for a particular type of physical problem. For the motion of an N-particle system in configuration space, it is most convenient to define the relative co-ordinates $\vec{Q}_2, \vec{Q}_3, \ldots, \vec{Q}_N$, so that the kinetic energy becomes

$$T = \frac{1}{2}[\dot{Q}_1^2 + \dot{Q}_2^2 + \dot{Q}_3^2 + \ldots + \dot{Q}_N^2]. \tag{2}$$

Here $^1/_2\dot{Q}_1^2$ is the kinetic energy of the center of mass, and $\vec{Q}_1$ itself is the square root of the mass of the system times the co-ordinate of the center of mass. If we let

$$M_k = m_1 + m_2 + m_3 + \ldots + m_k, \tag{3}$$

then

$$\vec{Q}_1 = (M_N)^{-1/2}\sum_i m_i \vec{r}_i. \tag{4}$$

In order to convert the kinetic energy from the form of equation (1) to that of equation (2), it is necessary to make the transformation

$$(m_i)^{1/2}\vec{r}_i = \sum_k S_{ki}\vec{Q}_k, \tag{5}$$

where the coefficients S_{ki} form a unitary matrix, so that

$$\sum S_{ki}S_{ji} = \delta_{jk} \tag{6}$$

and

$$\vec{Q}_k = \sum_i S_{ki}(m_i)^{1/2}\vec{r}_i. \tag{7}$$

The matrix components $S_{i1}, S_{i2}, \ldots, S_{iN}$ form the components of an N-dimensional vector $\vec{S}_i$. From equation (6) it follows that the N vectors $\vec{S}_1, \vec{S}_2, \ldots, \vec{S}_N$ have unit length and are orthogonal to each other. Thus the $\vec{S}_1, \vec{S}_2, \ldots, \vec{S}_N$ form a set of co-ordinate axes in N-space and are uniquely determined except for a rotation of the co-ordinate axes in N-space or a similarity transformation. One possible choice of the $\vec{S}_i$ is obtained in the following manner. Comparing equation (4) with equation (7), it follows that

$$S_{1j} = \left(\frac{m_j}{M_N}\right)^{1/2}, \qquad 1 \leqslant j \leqslant N. \tag{8}$$

The remainder of the S_{kj} can be determined from the Schmidt orthogonalization process. The coefficients S_{2j} are determined by requiring that $\vec{Q}_2$ be a linear combination of $\vec{r}_1$ and $\vec{r}_2$ such that $\vec{S}_2$ is orthogonal to $\vec{S}_1$; the S_{3j} are determined by requiring that $\vec{Q}_3$ be a linear combination of $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$ such that $\vec{S}_3$ is orthogonal to both $\vec{S}_1$ and $\vec{S}_2$. Thus each successive $\vec{Q}_k$ involves the co-ordinates of one more particle and is constructed so that $\vec{S}_k$ is orthogonal to each of the previous $\vec{S}_1, \vec{S}_2, \ldots, \vec{S}_{k-1}$. In this manner we obtain, for $2 \leqslant k \leqslant N$,

$$S_{kj} = -\left(\frac{m_k m_j}{M_k M_{k-1}}\right)^{1/2}, \qquad j < k, \tag{9}$$

$$S_{kk} = \left(\frac{M_{k-1}}{M_k}\right)^{1/2}, \qquad j = k, \tag{10}$$

$$S_{kj} = 0, \qquad j > k. \tag{11}$$

Accordingly, we can form the relative co-ordinates

$$\vec{Q}_2 = \left(\frac{m_1 m_2}{M_2}\right)^{1/2}(\vec{r}_2 - \vec{r}_1), \tag{12}$$

$$\vec{Q}_3 = \left(\frac{m_3}{M_2 M_3}\right)^{1/2}(m_1(\vec{r}_3 - \vec{r}_1) + m_2(\vec{r}_3 - \vec{r}_2)), \tag{13}$$

. . . . . .

$$\vec{Q}_k = \left(\frac{m_k}{M_{k-1}M_k}\right)^{1/2}\sum_{j=1}^{k-1} m_j(\vec{r}_k - \vec{r}_j). \tag{14}$$

Also, from equation (5), the $\vec{r}_i$ are given explicitly in terms of the $\vec{Q}_k$.

The co-ordinates $\vec{Q}_2$ are familiar in respect to the solution of two-body problems. Eyring, Polanyi, and Wigner[1] introduced the coordinates $\vec{Q}_2$ and $\vec{Q}_3$ in order to

consider the relative motion of three hydrogen atoms. They formed the "skewed" co-ordinate system in which the potential energy surface was plotted. Our set of relative co-ordinates generalizes the "skewed co-ordinates" to systems of N particles. These co-ordinates should be useful in molecular collision problems and molecular rotation-vibration problems. Their principal use, however, will probably lie in the prediction of the variation of chemical reaction rates with isotopic masses, since the zero-point energy in the activated complex and also the transmission coefficient depend rather sensitively on the masses which appear in the relative co-ordinates.

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[1] H. Eyring and M. Polanyi, *Z. Physik. Chem.*, *B*, **12**, 279, 1931; J. O. Hirschfelder, dissertation Princeton University, 1935; S. Glasstone, K. Laidler, and H. Eyring, *Theory of Rate Process* (New York: McGraw-Hill Book Co., Inc., 1941), p. 100.

RADIOACTIVE STRONTIUM FALLOUT

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CONVERSION FACTORS

"Average soil"	= 20 gm Ca/ft² in top 2.5 inches
1 MPC unit in any medium	= 1 μc Sr^{90}/kg Ca
	= 2,200 dpm Sr^{90}/gm Ca
1 megaton fission distributed uniformly over entire earth	= 0.0009 MPC unit
	= 0.5 mc Sr^{90}/mi²

I. EXPERIMENTAL MEASUREMENTS

Strontium 90 is of particular importance among the fission products because of chemical and physical characteristics which result in comparatively high retention in the skeleton. These are chemical similarity to Ca, an element essential to both plants and animals; an average life of about 40 years; and a low rate of elimination from the skeleton. On the basis of studies of the comparative effects of Sr^{90} and Ra^{226} in experimental animals and of the effects of Ra in humans, the generally accepted maximum permissible body burden (skeletal content) of Sr^{90} in adult humans is 1 microcurie. Since the body of the average adult contains about 1,000 gm. of Ca, this is equivalent to saying that the maximum permissible average concentration of Sr^{90} in the adult skeleton is 1 μc/1,000 gm. of Ca. For purposes

of this discussion, this ratio of Sr^{90} to Ca, in whatever medium it may occur, is designated an "MPC unit." One MPC unit of Sr^{90} in the human body is considered to be safe—a significant risk occurring only at much higher dosages.[1] The majority of analyses for Sr^{90} encountered in this work were of the order of a few thousandths of 1 MPC unit. For purposes of orientation, it is helpful to remember that 0.001 MPC unit corresponds to 1/1,000 μc of Sr^{90}/kg of Ca, or 2.2 dpm of Sr^{90}/gm of Ca. The small weights of Sr^{90} involved in both the radiostrontium and normal strontium being considered, and the similarities of the element to Ca, justify the assumption that its distribution in the body will follow that of Ca in a general way.

Two megatons of fission will produce 1 millicurie (mc.) of Sr^{90}/mi^2, if the fission products are uniformly distributed over the earth's surface. If this amount of radioactivity is mixed with the available Ca in the soil, an average of about 20 gm/ft^2 in the top 2.5 inches of soil, the specific radioactivity produced is 0.0018 MPC unit. It is observed that most of the Sr^{90} fallout is concentrated in the top 1 or 2 inches of soil. For example, in Tables 1 and 2, which show the Sr^{90} burden in the fall of 1953 in the soil of twelve farms in the Wisconsin-Illinois area as well as in the alfalfa and the milk of the cows fed thereon, we note that the top inch of soil contains about 56 per cent and the next 5 inches contain the remaining 44 per cent of the total Sr^{90}. Recently some evidence has been discovered that the radiostrontium finds its way to greater depths.[2] In Table 2 data are given for Iowa soil collected in 1937 which, as expected, shows no Sr^{90}. The average available Ca content of the domestic soils was 8 ± 1 gm Ca/ft^2/in., the average fraction of total Ca exchangeable was 68 ± 3 per cent, and the average Sr^{90} content was 4.7 ± 0.4 mc/mi^2.

TABLE 1

BIOSPHERE Sr^{90} ASSAYS* (WISCONSIN MILKSHED—PRE-CASTLE, OCTOBER, 1953)
(Values Are Given in Terms of 0.001 MPC Units, Except where Noted)

	Soil		Total Sr^{90}		
Farm†	0″–1″	1″–6″	(Mc/Mi^2)	Alfalfa	Milk
Grabow, Wisconsin	26.2	6.7	4.5	12.8	1.7
Oliver Swain, Wisconsin	7.4	2.2	3.1	5.3	1.3
Swanson, Illinois	15.8	2.5	9.2	7.1	1.2
Holcomb, Wisconsin	8.7	1.8	5.1	8.3	1.6
Lewke, Wisconsin	10.2	2.9	3.5	20.9	2.3
Premo, Wisconsin	13.1	2.5	3.8	4.1	0.7
Kurpeski, Illinois	16.3	5.6	4.0	7.4	1.3
Austin, Illinois	22.4	4.7	4.7	5.0	1.8
McKee, Illinois	8.1	0.9	6.3	14.8	1.4
Blomberg, Illinois	1.7	<0.3	4	9.5	1.2
Van Winkle, Illinois	13.8	7.9	3.8	5.0	...
Carver, Illinois	42.1	5.6	3.3	2.3	...
Average	16.8	3.9	...	8.9	1.4
Average Sr^{90} (mc/mi^2)	2.6	2.1	4.7 ± 0.4		
Total Sr^{90} (mc/mi^2)		4.7 ± 0.4			

* E. A. Martell and W. F. Libby, Project Sunshine Bull. No. 10, January 10, 1955; E. A. Martell, *ibid.*, Suppl. 3, September 1, 1955.
† Samples collected by Dr. Lyle T. Alexander, Chief, Soil Survey Laboratory, Plant Industry Station, Beltsville, Maryland.

As might be expected because of the similarity of the Sr chemistry to that of Ca, milk and cheese show radiostrontium without exception. Figures 1*a*, 1*b*, 1*c*, and 1*d* show the data for both foreign and domestic samples.

TABLE 2*

1953 Domestic Pre-Castle Soil Samples

Location	Lab. No.	Date Sample Taken	Area of Sample ($Ft.^2$)	Depth of Sample (Inches)	Ca Extracted (1 *N* NH_4Ac Reflux) (Gm.)	Weight Sample Extracted (Pounds)	Exchangeable Ca, Analytical Method (Me/100 Gm)	Total Weight Sample Taken (Pounds)	Calc. Exch. Ca in Sample (Gm.)	Calc. Ca. (Gm/Ft^2)	Per Cent Ca. Extracted (10th Col.)	Milli-MPC Units	Total Sr^{90} (Mc/Mi^2)	
Rock Co., Wis., Site No. 1, Knox fine sandy loam, Grabow Farm	531665	9/28/53	1.5	0–1	3.2	10.0	4.8	11.0	4.8	3.2	72.7	26.2	2.3	
Rock Co. Wis. Site No. 1, Knox fine sandy loam	531665	9/28/53	1.5	0–1	1.0	10	4.8	11.0	4.8	3.2	(6 *N* HCl after NH_4Ac extract)	24.6	...	4.5
Rock Co., Wis. Site No. 1, Knox fine sandy loam	531666	9/28/53	0.3	1–6	3.0	10	3.5	11.0	3.5	11.7	93.8	6.7	2.2	
Rock Co., Wis., Swain Farm, Site No. 2, Knox fine sandy loam	531667	9/28/53	1.4	0–1	10.8	9.5	8.5	10.5	8.1	5.8	147	7.4	1.2	
Rock Co., Wis., Swain Farm, Site No. 2, Knox fine sandy loam	531668	9/29/53	0.3	1–6	7.4	9.5	9.8	10.5	9.3	31.0	88.1	2.2	1.9	3.1
Winnebago Co., Illinois, Swanson Farm, Site No. 3, Carrington lib. sil	531669	9/29/53	1.2	0–2	9.5	6.0	13.9	7.0	8.8	7.3	125	15.8	3.2	
Winnebago Co., Illinois, Swanson Farm, Site No. 3, Carrington lib. sil	531670	9/29/53	0.24	1–6	10.0	6.5	13.5	7.5	9.2	38.3	125	2.5	2.7	5.9
Rock Co., Wis., Holcomb Farm, Site No. 4, Carrington sil	531671	9/29/53	1.33	0–1	9.4	6.5	14.9	7.5	10.1	7.6	107	8.7	1.8	
Rock Co., Wis., Holcomb Farm, Site No. 4, Carrington sil	531672	9/29/53	0.36	1–6	11.6	8	14.3	9.0	11.7	32.5	111	1.8	1.6	3.4
Dave Co., Wis., Lewke Farm, Miami sil, Site No. 4	531673	9/30/53	1.2	0–1	5.7	7	7.7	8.0	5.6	4.7	116	10.2	1.3	
Dave Co., Wis., Lewke Farm, Miami sil, Site No. 4	531674	9/30/53	0.28	0–6	7.0	8.5	8.8	9.5	7.6	27.1	103	2.9	2.2	3.5
Columbia Co., Wis., Premo Farm, Miami sil, Site No. 6	531675	9/30/53	1.2	0–1	5.0	7.5	10.2	8.5	7.9	6.6	72.4	13.1	2.4	
Columbia Co., Wis., Premo Farm, Miami sil, Site No. 6	531676	9/30/53	1.2	0–1	2.0	7.5	10.2	8.5	7.9	6.6	...	12.5	...	3.8
Columbia Co., Wis., Premo Farm, Miami sil, Site No. 6	531676	9/30/53	0.30	1–6	8.2	9.0	6.7	10.0	6.1	20.3	149	2.5	1.4	
McHenry Co., Illinois, Miami sil, Kurpeski Farm, Site No. 7	531677	9/30/53	1.16	0–1	5.0	7.0	6.4	8.0	4.6	4.0	122	16.3	1.8	
McHenry Co., Illinois, Miami sil, Kurpeski Farm, Site No. 7	531678	9/30/53	0.30	1–6	5.0	9.0	4.7	10.0	4.3	14.3	132	5.6	2.2	4.0
McHenry Co., Illinois, Miami sil, Austin Farm, Site No. 8	531679	10/1/53	1.3	0–1	4.2	8.0	6.8	9.0	5.6	4.3	85.7	22.4	2.7	
McHenry Co., Illinois, Miami sil, Austin Farm, Site No. 8	531680	10/1/53	0.31	1–6	3.9	9.5	5.1	10.5	4.9	15.8	88.6	4.7	2.0	4.7

TABLE 2—*Continued*

Location	Lab. No.	Date Sample Taken	Area of Sample (Ft.²)	Depth of Sample (Inches)	Ca Extracted (1 *N* NH_4Ac Reflux) (Gm.)	Weight Sample Extracted (Pounds)	Exchangeable Ca, Analytical Method (Me/100 Gm)	Total Weight Sample Taken (Pounds)	Calc. Exch. Ca in Sample (Gm.)	Calc. Ca. (Gm/Ft²)	Per Cent Ca Extracted (10th Col.)	Milli-MPC Units	Total Sr^{90} (Mc/Mi²)	
McHenry Co., Illinois, McKee Bros. Farm, Site No. 9, Drummer rich	531681	10/1/53	1.14	0–1	16.0	7.0	25.9	8.0	18.8	16.5	97.0	8.1	3.7	
McHenry Co., Illinois, McKee Bros. Farm, Site No. 9, Drummer rich	531681	10/1/53	1.14	0–1	10.0	7.0	25.9	8.0	18.8	16.5	(6 *N* HCl after NH_4Ac extract)	4.0	...	6.3
McHenry Co., Illinois, McKee Bros. Farm, Site No. 9, Drummer rich	531682	10/1/53	0.24	1–6	23.5	9.0	27.4	10.0	24.9	103.8	105	0.9	2.6	
McHenry Co., Illinois, Blomberg Farm, Site No. 10, Drummer rich	531683	10/1/53	0.85	0–1	8.0	5.0	25.6	6.0	13.9	16.4	69.0	1.65	(0.8)	
McHenry Co., Illinois, Blomberg Farm, Site No. 10, Drummer rich	531683	10/1/53	0.85	0–1	5.0	5.0	25.6	6.0	13.9	16.4	(6 *N* HCl extraction following NH_4Ac)	4.4	2.0	
McHenry Co., Illinois, Blomberg Farm, Site No. 10, Drummer rich	531684	10/1/53	0.22	1–6	17.5	8.0	25.4	9.0	20.8	94.5	95.1	0.3	...	
Will Co., Illinois, Van Winkle Farm, Plainfield sand, Site No. 11	531685	10/2/53	1.23	0–1	3.1	9.0	3.5	10.0	3.2	2.6	107	13.8	1.0	
Will Co., Illinois, Van Winkle Farm, Plainfield sand, Site No. 11	531686	10/2/53	0.26	1–6	3.8	10.5	3.2	11.5	3.3	12.7	127	7.9	2.8	3.8
Will Co., Illinois, Carver Farm, Site No. 12, Plainfield sand	531687	10/2/53	1.38	0–1	2.1	9.0	2.8	10	2.5	1.8	91.3	42.1	2.1	
Will Co., Illinois, Carver Farm, Site No. 12, Plainfield sand	531688	10/2/53	0.35	1–6	2.2	11.0	2.4	12	2.6	7.5	91.7	5.6	1.2	3.3
Utah, College Pasture	531689	10/53	1.45	0–1	11.2	7.5	28.0	7.5	19.1	13.2	58.6	1.38	0.51	
Utah, College Pasture	531690	10/53	0.265	1–6	13.2	8.5	28.8	8.5	22.2	83.4	59.4	0.20	0.47	0.98

Average Ca content (exchangeable): 8.0 ± 1 gm $Ca/ft^2/in$ = 8.6 ± 1 mg $Ca/cm^2/in$.
Average fraction of total Ca exchangeable = 68 ± 3 per cent (assuming 6 *N* HCl to extract all Ca).

$$\frac{\text{Sr}^{90}\text{ concentration, 1–6 inches}}{\text{Sr}^{90}\text{ concentration, 0–1 inches}} = 0.23 \pm 0.027.$$

Effective depth at top 1 inch concentration = 2.15 inches = 19 mg Ca/cm^2 = 18 gm Ca/ft^2.

Pre-atomic samples:

Iowa, Carrington loam	C-2916	1937	0–3	0 ± 0.00005 MPC units
Iowa, Carrington loam	C-2917	1937	3–13	0 ± 0.00005 MPC units

* All radiostrontium measurements made in Chicago Sunshine Laboratory (see E. A. Martell and W. F. Libby, Project Sunshine Bull. No. 10, January 10, 1955; E. A. Martell, *ibid* Suppl. 1, March 1, 1955; Suppl. 2, June 1, 1955; Suppl. 3, September 1, 1955; and E. A. Martell, Project Sunshine Bulletin 11, December 1, 1955).

The amount of radiostrontium found in humans is shown in Figures 2*a* and 2*b*. The data show that the present Sr^{90} content probably averages somewhat less than 0.001 MPC unit in young people. Apparently a number of barriers protect the human skeleton from this fallout radioactivity.

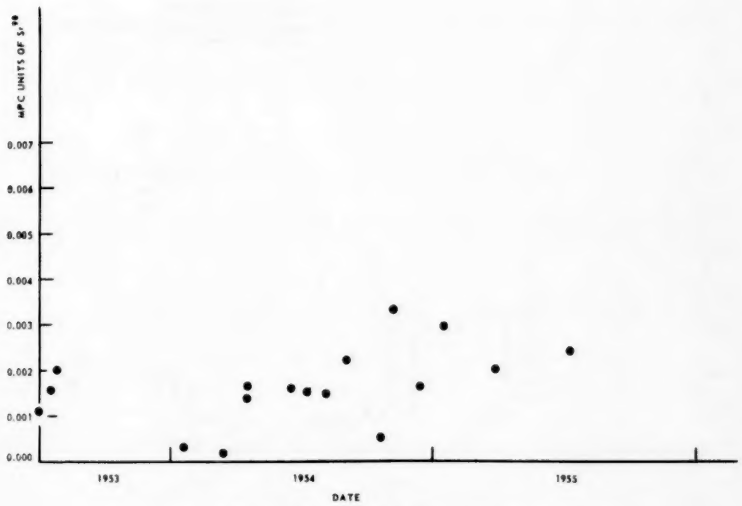

FIG. 1*a*.—Wisconsin cheese—Sr^{90} content

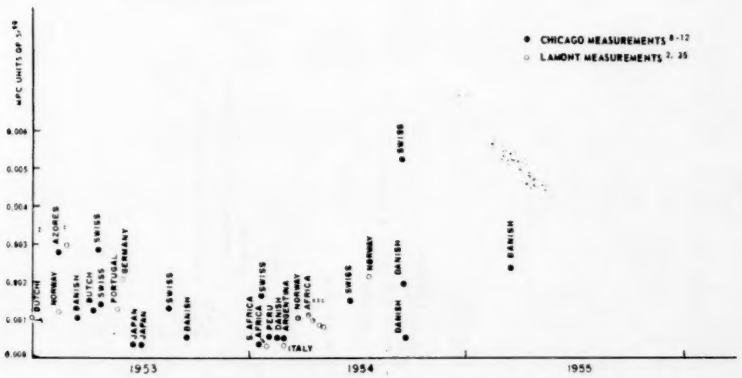

FIG. 1*b*.—Foreign cheese—Sr^{90} content

Measurements have been made on animals, principally cattle and sheep. These data are given in Figures 3*a* and 3*b*. We see here that the contents are much higher than those for milk and human samples, apparently due to selective deposition of Sr in the animal bones, which protects the milk and thus human bone.

Data for foreign soil samples collected just before the Castle test series are presented in Table 3 and Figures 4*a* and 4*b*. From these data we deduce that the band around the earth bounded by the latitudes 60° N. and 10° S. shows a deposi-

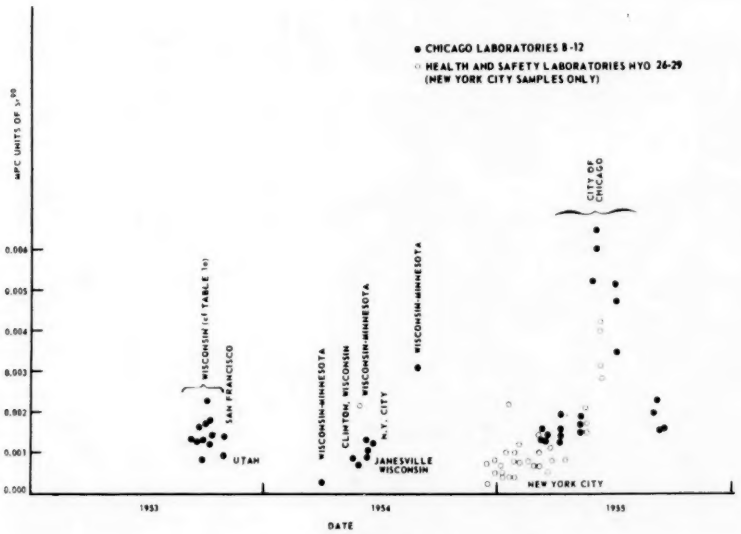

FIG. 1*c*.—U.S. milks—Sr^{90} content

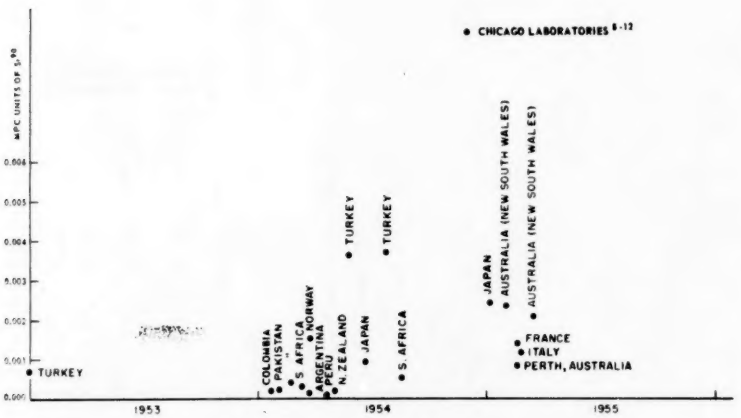

FIG. 1*d*.—Foreign milks—Sr^{90} content

tion of 0.8 megatons (MT) equivalent of Sr^{90}, in addition to some 0.4 MT which appears to be nearly uniformly deposited, as would have been expected from a slow deposition from a large stratospheric reservoir. No evidence for longitudinal variation is apparent in Figure 4*b*.

TABLE 3*

Sr^{90} Content and Other Pertinent Data on Foreign Soil Samples Collected before Castle

Lat./Long.	Location	Lab. No.	Date Sample Taken	Area of Sample (Ft^2)	Depth of Sample (Inches)	Ca Extracted (1 N NH_4Ac Extraction (Gm.)	Weight Sample Extracted (Pounds)	Exchangeable Ca, Analytical Method (Me/100 Gm.)	Total Weight Sample Taken (Pounds)	Calc. Exch. Ca in Sample (Gm.)	Calc. Exch. Ca (Gm/Ft^2)	(Gm/Ft^2/In.)	Per Cent Ca Extracted (7th Col.)	Sr^{90} Content (Milli-MPC Units)	Sr^{90} Fallout (Mc/Mi^2)
35° S./60° W.	Buenos Aires, Arg. (8 miles apart)	54569	3/15/54	2	0–4	8.7	8	15.0	45.7	62.2	31.1	7.8	80	0.45 ± 0.03	0.39
35° S./60° W.	Buenos Aires, Arg.	54570	3/15/54	2	0–4	6.6	8	10.2	45.7	42.3	21.2	5.3	89	0.44 ± 0.03	0.26
2° N./104° E.	Bin Tong Park, Sing.	54571	3/4/54	2	0–4	0.01	8	0.4	40.6	1.5	0.73	0.13	3	≤ 50	≤1.0
2° N./104° E.	Tengah Air Base, Sing.	54572	3/4/54	2	0–4	0.3	8	0.4	42.5	1.54	0.77	0.19	100	22 ± 2	0.5
30° N./75° E.	Molir Village, Pakistan	54671	2/25/54	2	0–4	4.3	4	18.4	60.5	101	50.5	12.6	64	0.27 ± 0.03	0.38
30° N./75° E.	Hub River, Pakistan	54672	2/25/54	2	0–4	4.0	4	16.6	65.3	98.4	49.2	12.3	66	0.35 ± 0.04	0.48
52° N./0°	Rothamstead, Eng.	54675	4/?/54	7 (est.)	0–3	4.5	4	13.5	95.8	117.4	16.8	5.6	92	1.31 ± 0.07	0.61
5° N./75° W.	Bogotá, Colombia (plowed)	54715	3/?/54	2	0–4	4.3	4	13.8	29.9	37.5	18.7	4.7	86	0.67 ± 0.04	0.35
5° N./75° W.	Bogotá, Colombia (grass)	54716	3/?/54	2	0–4	4.8	4	13.9	51.0	64.4	32.2	8.1	95	0.91 ± 0.08	0.82
5° S./30° E.	Leopoldville, Belgian Congo	54717	3/?/54	2	0–4	2.4	4	4.6	51.1	21.3	10.7	2.7	All	0.92 ± 0.08	0.27
5° S./30° E.	Leopoldville, Belgian Congo	54718	3/?/54	2	0–4	4.5	4	13.2	69.4	83.2	41.6	10.4	94	0.21 ± 0.04	0.24
12° N./45° E.	Aden Protectorate, SW Arabia	54421	3/?/54	2	0–4	9.6	8	23.7	32.0	68.9	34.4	8.6	56	0.51 ± 0.03	0.49
12° N./45° E.	Aden Protectorate, SW Arabia	54422	3/?/54	2	0–4	9.4	8	19.7	30.0	53.7	26.8	6.7	66	1.05 ± 0.10	0.78
60° N./10° E.	Oslo, Norway (Walsh)	54412	4/27/54	2.21	0–2	9.8	8	17.0	16.2	25.0	11.3	5.7	79	1.51 ± 0.05	0.48
60° N./10° E.	Oslo, Norway, No. 1	54410	4/27/54	2.21	0–2	11.0	8	18.0	16.0	26.2	11.9	5.9	84	1.44 ± 0.06	0.48
32° N./36° E.	Beka'a Valley, Lebanon	54293	2/25/54	1.5	0–3	20.2	7	41.5	17.0	64.1	42.7	14.2	77	0.86 ± 0.05	1.0
35° N./2° E.	Boghari, Algeria, No. 3	54359	2/22/54	2.21	0–2	13.2	8	29.8	19.0	51.4	23.3	11.6	61	3.4 ± 0.08	2.2
35° N./2° E.	Boghari, Algeria, No. 1	54360	2/22/54	2.76	0–2	13.7	8	34.3	19.4	60.4	21.9	10.9	55	3.8 ± 0.1	2.3
35° N./2° E.	Boghari, Algeria, No. 2	54361	2/22/54	2.21	0–2	10.0	8	25.8	18.3	42.5	19.4	9.7	53	3.46 ± 0.08	1.9
35° N./40° E.	Village 41 km. SE. of Damascus, Syria	54295	2/26/54	2.21	0–2	12.4	7	27.1	14.4	35.4	15.6	7.8	72	1.9 ± 0.08	0.82
35° N./40° E.	Tel Muskan, Syria	54296	2/26/54	2.21	0–2	18.6	7	29.6	20.0	53.8	24.3	12.1	99	1.1 ± 0.10	0.75
52° N./0°	Ffostill, Breton, Wales	54415	4/?/54	2.78	0–2	8.0	8	11.4	11.4	11.8	4.27	2.1	97	3.3 ± 0.12	0.39
52° N./0°	Montgomery, B.I., Wales	54416	4/?/54	2.78	0–2	9.6	8	11.6	8.3	8.7	3.1	1.6	All	...	...
52° N./5° W.	Cardigan, B.I., Wales	54417	4/?/54	2.78	0–2	0.3	5	1.3	10.2	1.2	0.4	0.2	50	97 ± 9.1	1.1
52° N./0°	Suffolk, Eng., No. 4	54418	4/?/54	1.74	0–2	15.2	7	37.2	16.6	56.1	32.2	16.1	64	1.37 ± 0.06	1.2
52° N./0°	Suffolk, Eng., Pudding Crnr.	54419	4/?/54	1.74	0–2	18.1	8	38.0	16.8	58.0	33.3	16.7	66	0.89 ± 0.05	0.83
52° N./0°	Suffolk, Eng., Old Orchard	54420	4/?/54	1.74	0–2	16.6	8	36.2	17.9	58.8	33.8	16.9	63	0.94 ± 0.09	0.89
45° S./170° E.	New Zealand, Hutt Co.	531804	11/?/53	5 (est.)	0–3	7.9	8.5	13.0	44.0	51.9	10.4	3.5	79	≤ 0.21	≤0.06
45° S./170° E.	New Zealand, S. Canterbury	5485	1/18/54	5 (est.)	0–3	6.5	9	10.8	47.2	46.3	9.3	3.1	74	≤ 0.18	≤0.05
45° S./170° E.	New Zealand, Whangarei Co.	5471	1/18/54	5 (est.)	0–3	0.9	8	1.7	37.7	5.8	1.20	0.4	75	2.51 ± 0.09	0.084
20° S./45° W.	Belo Horizonte, Brazil (3 km. apart)	54288	3/?/54	2	0–4	0.2	10	0.4	42.0	1.5	0.8	0.2	50	13.5 ± 1.25	0.30
20° S./45° W.	Belo Horizonte, Brazil	54289	3/?/54	2	0–4	0.6	8	0.7	26.5	1.7	0.9	0.2	All	4.17 ± 0.3	0.10
35° S./75° W.	Santiago, Chile	5472	12/?/53	30	0–1	10.2	8	27.2	50.4	124.5	4.15	4.1	51.5	≤ 0.33	≤0.04
30° S./30° E.	Natal, S. Africa	54399	2/?/54	2	0–4	2.6	8	4.2	29.5	11.2	5.6	1.4	86.7	0.49 ± 0.08	0.076
30° S./30° E.	Natal, S. Africa, 3 mi. SE.	54400	2/?/54	2	0–4	1.7	8	1.2	47.0	5.1	2.55	0.6	...	9.80 ± 0.71	0.70
15° N./120° E.	Philippine Islands	54401	2/26/54	2	0–4	14.8	8	18.6	48.4	81.7	40.8	10.2	All	1.47 ± 0.21	1.67
15° N./120° E.	Philippine Islands	54402	2/26/54	2	0–4	3.5	8	5.6	51.7	26.3	13.1	3.3	85.4	20.1 ± 2.3	7.3

* See note to Table 2.

Average: 8.1 ± 0.8 gm Ca/ft^2/in = 8.8 ± 0.9 mg Ca/cm^2/in.

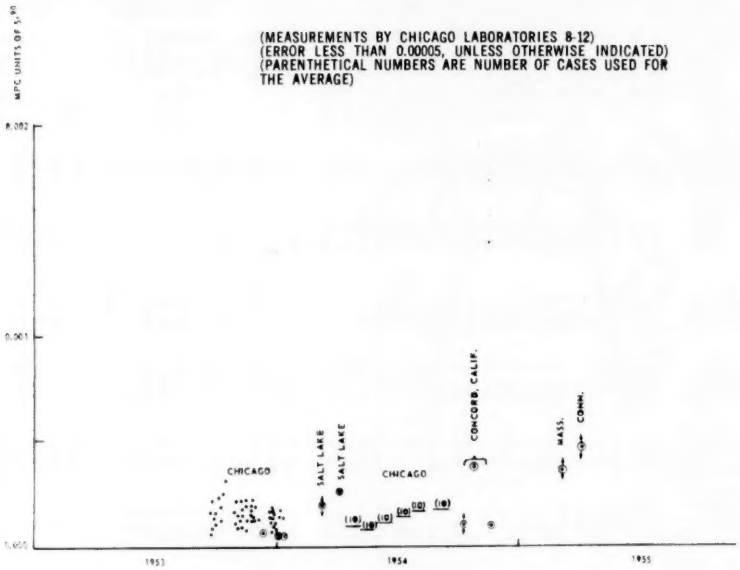

FIG. 2*a*.—Human stillborns

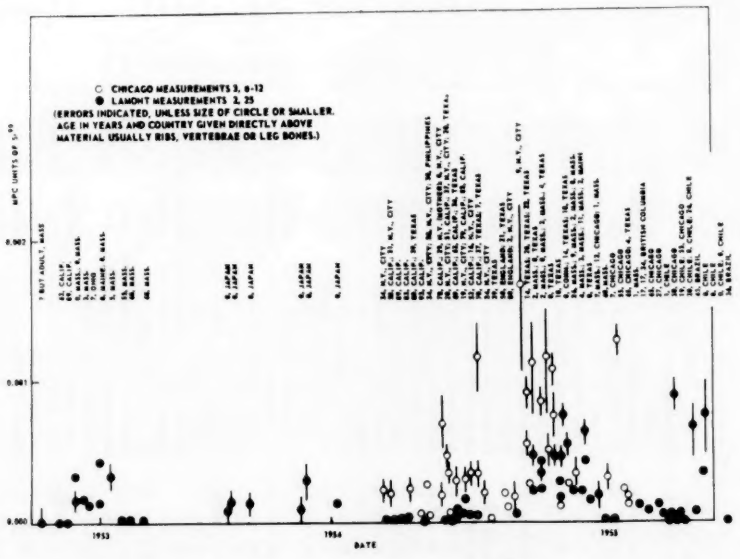

FIG. 2*b*.—Human bone—Sr^{90} content

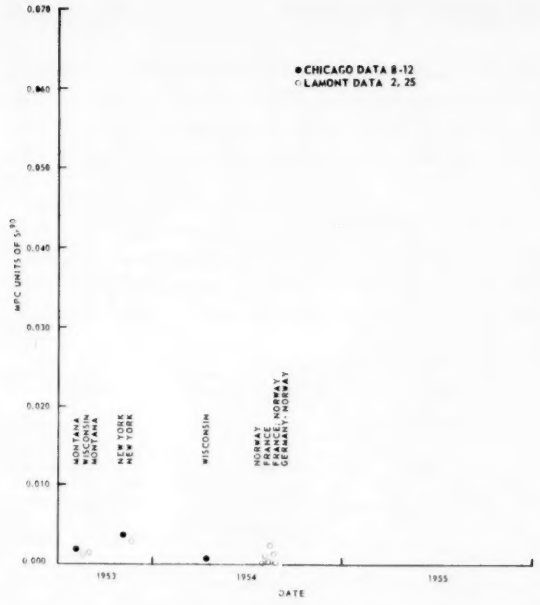

FIG. 3*a*.—Calf bones

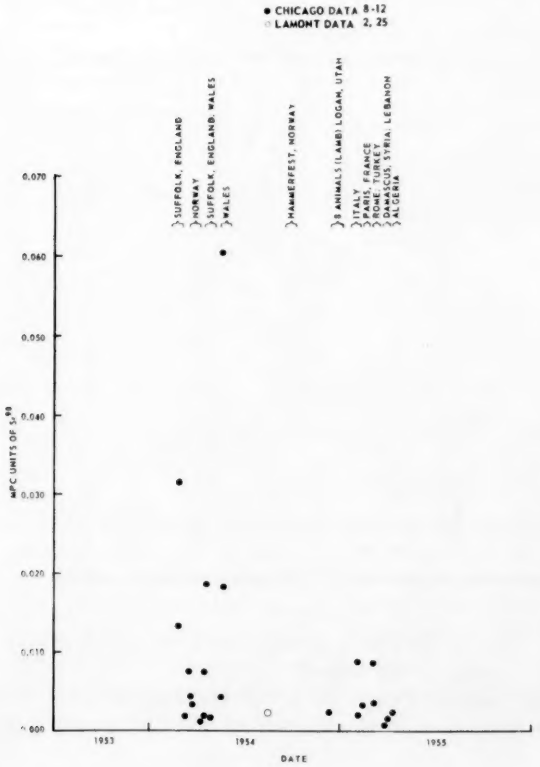

FIG. 3*b*.—Sheep bones

The actual heights of rise of the bomb clouds are the basis for the assumption that all distant fallout from megaton weapons occurs from a stratospheric reservoir, while that from those of lower yield occurs from the troposphere. Actually, the height of the tropopause varies with the season, so that the season must be con-

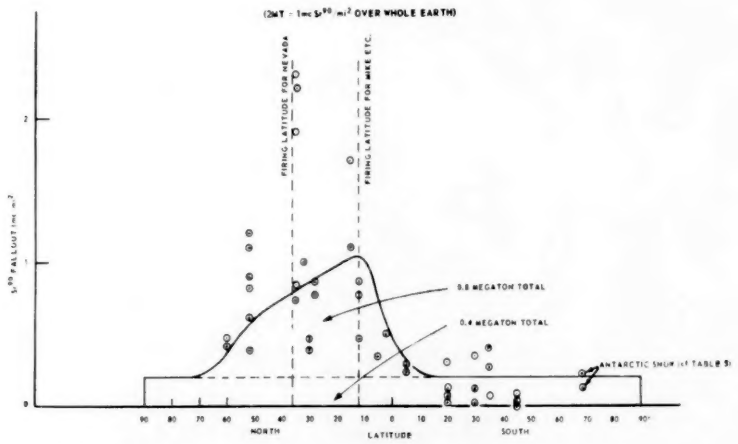

FIG. 4*a*.—Latitudinal distribution of foreign pre-Castle Sr^{90} fallout (soil assays)

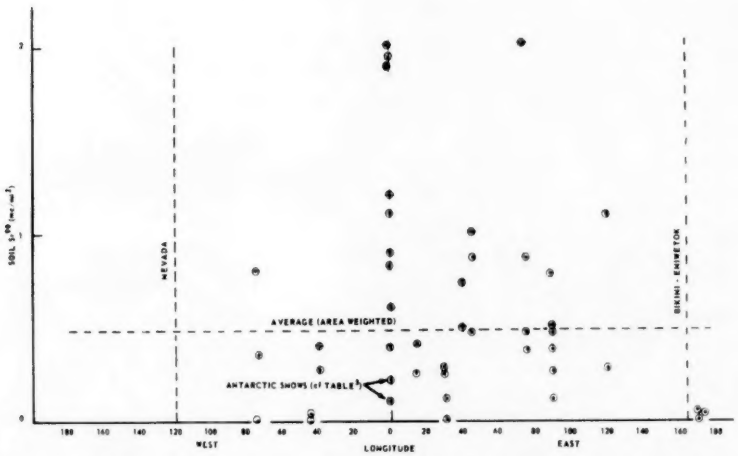

FIG. 4*b*.—Longitudinal distribution of pre-Castle Sr^{90} soil data from foreign samples.

sidered in the assignment. During the Pacific tests it has been near 55,000 ft.; hence our classification has validity in this respect.

Two of the most important data in Figures 4*a* and 4*b* are those from the antarctic series. The samples were snow cores, collected for the Chicago Sunshine

Laboratory and for cosmic-ray tritium analysis,[3-5] by Mr. Paul Humphrey of the United States Weather Bureau in January and February, 1955, at Admiral Byrd Bay (69°34′ S.; 00°41′ W.), at Atka Bay (70°35′ S.; 08°06′ W.), and at Little America III. The data are given in Tables 4 and 5. Table 4, Part A, gives the Sr^{90} and T contents of surface snow at four locations. From the T concentrations and the expected T production rate in this region[3-5] (T produced in the Castle test itself was precipitated out in a few weeks and never entered the Southern Hemisphere appreciably, because of the large amount of water taken into the cloud with which it became mixed[6]), we determined the annual precipitation to be 7.8 ± 2 inches of water. This calculation, together with that for the January–February,

TABLE 4

POST-CASTLE Sr^{90} FALLOUT IN ANTARCTICA

A. SURFACE SNOW

Date	Location	Sr^{90} (dpm/Liter)	T (Atoms/10^{18} H's)
January 15, 1955	Near Quonset, Little America III	3.2 ± 0.3	14.1 ± 0.6
January 17, 1955	One-half mile east, Little America III	3.1 ± 0.7	7.5 ± 0.6
February, 1955	Atka Bay, 6 miles inland on shelf (70°35′ S.; 08°06′ W.)	5.3 ± 0.5	19.2 ± 0.8
February 19, 1955	Admiral Byrd Bay (69°34′ S.; 00°41′ W.)	2.0 ± 0.2	24 ± 5
	Average	3.4 ± 0.5	14 ± 3

B. Sr^{90} PRECIPITATION RATE IN JANUARY AND FEBRUARY, 1955

Annual snow precipitation rate from T assay* and 0.59 T's/cm²/sec as the expected antarctic cosmic-ray T production.

$$\rho = \frac{4.7 \times 0.59}{14} \text{ meters/yr} = 7.8 \pm 2 \text{ inches of water.}$$

Sr^{90} rate of precipitation:

3.4 ± 0.5 dpm/liter = 62 dpm/ft²/yr for 7.8 inches of water/yr = 0.8 ± 0.2 mc/mi²/yr.

* H. von Buttlar and W. F. Libby, "Natural Distribution of Cosmic Ray Produced Tritium. II," *J. Inorg. and Nuclear Chem.*, **1**, 75, 1955; L. A. Currie, W. F. Libby, and R. Wolfgang, *Phys. Rev.*, **101**, 1557, 1956.

TABLE 5

PRE- AND POST-CASTLE Sr^{90} FALLOUT AT ADMIRAL BYRD BAY (69°34′ S.; 00°41′ W.)
(Collected 2/19/55)

SNOW CORE

Age (Years) (7.8 Inches Water/Yr)	Depth (Ft.)	Density	Sr^{90} (dpm/Liter)	T (Atoms/10^{18} H's)	Sr^{90} Rate* (Mc/Mi²/Yr)
0–0.54	0–1	0.35	1.95 ± 0.20	24 ± 5	0.46
0.54–1.04	1–2	0.32	1.7 ± 0.2	12.5 ± 0.8	0.40
1.04–1.52	2–3	0.30	0.48 ± 0.04	13.5 ± 0.7	0.11
1.52–2.16	3–4	0.41	0.90 ± 0.06	...	0.21

* Assumed annual precipitation, 7.8 inches water/yr, on the basis of T contents of surface waters (cf. Table 4).

1955, Sr^{90} precipitation rate of 0.8 mc/mi²/yr, are given in Part B of Table 4. The result for the annual precipitation agrees well with direct observation by the Atka Expedition and by Mr. Humphrey personally.[7] At Little America IV, direct observation showed that the floor of the tent projecting from the ice front was beneath only 7 or 8 feet of snow after roughly seven years. Since the density of the snow was about 0.35, this would correspond to about 5 inches of annual precipitation. Other observations[7] checked this general magnitude. In Table 5 a core taken at Admiral Byrd Bay was measured for both Sr^{90} and T. From these data we observe the pre-Castle fallout rates of 0.11 and 0.21 Sr^{90} mc/mi²/yr. The surface rate at this site is 0.43, which is less than the general average in the area for January and February, 1955, of 0.8 mc/mi²/yr—as shown in Table 4—hence it

may be that the pre-Castle values at this site are low also and should be increased by the ratio 0.8/0.43, or by 90 per cent, to 0.2 and 0.4, respectively.

The average Sr^{90} content of rain and snow in the Chicago area since the fall of 1952 was calculated by weighting each datum by the total rainfall observed in the particular storm. The data on the Sr^{90} content of Chicago rain are given in Figure 5.[8-12] It is clear that large fluctuations can occur in individual storms. However, these extremes were, in general, of low total rainfall; hence the effect on the average is small. It is interesting that the antarctic snows have about the same Sr^{90} content as the average Chicago rain of Figure 5. We recall that pre-

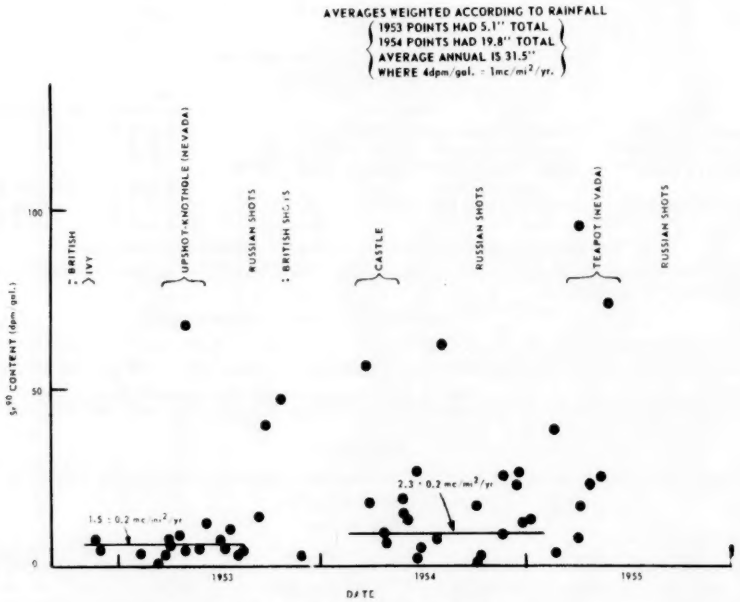

FIG. 5.—Sr^{90} content of Chicago rains

cipitation there is only one-fourth to one-fifth that in Chicago. According to the mechanism espoused in this paper, these fluctuations are to be expected, because the fallout from the stratosphere is thought to be steady and continuing, and the washing-out by rain is expected to carry down the fallout accumulated since the last precipitation from the particular air mass involved. Many of the samples given in Figure 5 and in Project Sunshine Bulletins Nos. 10 and 11[8-12] have been measured for T as well;[3-5] hence further correlations of the type described above for antarctic snow (Tables 4 and 5) can be made.

In Tables 1 and 2 the Sr^{90} contents of soils in the midwest region of the United States were shown to have an assay of 4.7 mc/mi² in October, 1953. The total from rains in the preceding year was only 1.5 mc/mi², according to Figure 5; hence we have to expect about 3 mc/mi² to have been deposited prior to Operation Ivy by direct fallout, most reasonably from tropospheric debris. The total fired in

Nevada prior to this time which would not have fallen out in the immediate vicinity of the test site was approximately 200 KT for the Operations Tumbler-Snapper and Buster-Jangle together. If this were all deposited in the United States, it would amount to about 7.0 mc/mi^2. The test series Upshot-Knothole, with approximately 220–230 KT of distant fallout, and Teapot, with approximately 160–180 KT, should have added correspondingly to the 4.7 mc/mi^2 in October, 1953, and to the 3.0 mc/mi^2 of stratospheric fallout for the intervening period, for a total of perhaps 12 mc/mi^2 expected in the spring of 1956 in the United States.

The efficiency with which rain removes fallout from the air through which it passes is probably high. One knows, on simple physical grounds, that as little as 0.1 inch of rain will traverse at least 90 per cent of the air volume lying below the layer in which the rain originates, so that 90 per cent of all particles which can be swept up by a falling raindrop will be carried down by such quantities of rain. On the point as to whether fallout is likely to be deposited by rain, we note that G. H. Wilkening[13] showed that the decay products of the radioactive gas Rn which in themselves are isotopes of nongaseous elements are found affixed to particles of diameters between 20 and 800 angstrom units (0.002–0.080 μ)—a submicroscopic range not at all unlikely for the radioactive fallout stored in the stratosphere.[1] The velocity of fall for such particles would be very small and in this respect quite compatible with the long stratospheric storage times indicated by the Project Sunshine data. Blifford, Lockhart, and Rosenstock[14] studied the concentration of the Rn decay products in rainfall in the Washington, D.C., area and concluded that rain was the mechanism by which the particles containing these products were precipitated and that the average time the decay products spent in the air before being precipitated was only 15 days, approximately. O. Haxel and G. Schumann,[15] found this time in Heidelberg, Germany, to be about 4 days, and Damon and Kuroda[16] concluded that Blifford *et al.*, were correct in attributing the precipitation of the aerosol carrying the Rn decay products to rain, their conclusion being based in part on additional data that they had taken. The average time spent by water in the air was found by von Buttlar and Libby[4] to be between 5 and 14 days.

For these reasons it seems very likely that rainfall or snowfall carries down a major part of the fallout which comes from the stratosphere and probably is a very important mechanism for that part of the tropospheric fallout material which does not fall out in the first few hours or day or two after the detonations. Of course, the whole question can be settled by direct experiment in which a correlation between rainfall and total fallout is sought. The present data seem to favor the hypothesis. This conclusion and prediction seem to be borne out by Table 6, which presents the total Sr^{90} content of the top 2 inches of typical United States soils collected in October, 1955, and leached at room temperature for 30 minutes with 6 *N* HCl.

Table 7 shows data obtained in the Chicago laboratories on the Sr^{90} contents of rivers and lakes. It is clear that these are much lower than those of the rain from which they are derived. For example, from Figure 5 we should estimate that the average rain in the Chicago area in the summer of 1953 had a Sr^{90} content of about 7 dpm/gal. From Table 1 for the domestic pre-Castle soil contents and from Figures 4*a* and 4*b* for foreign pre-Castle soil contents, we estimate that the European

rains averaged about 3 dpm/gal. The four rivers, Mississippi, Mosel, Seine, and Danube, show less than 5 per cent of this; hence we conclude that the Sr^{90} in rain is removed by the soil before the water runs off to the rivers and lakes. This fact agrees, of course, with the sharp localization of the Sr^{90} in the top 2 inches of soil (cf. Tables 1, 2, and 3).

TABLE 6*

Sr^{90} FALLOUT ACCUMULATION IN TOP SOIL (0–2 INCHES) IN U.S. IN 1955
(Sampled September 23–October 20, 1955)

Station	Measured in Soil (dpm/Ft²)
La Guardia	310†
	350 ± 21
	550 ± 16
Binghamton	710 ± 16
Philadelphia	450 ± 19
Rochester	550 ± 20
Jacksonville	470 ± 20
Atlanta	530 ± 12
Detroit	640 ± 21
New Orleans	470 ± 14
Memphis	900 ± 20
Des Moines	540 ± 13
Rapid City	1070 ± 21
Seattle	400 ± 15
Boise	1160 ± 23
Albuquerque	270 ± 14
	290 ± 20
Grand Junction	280 ± 14
Salt Lake City	860 ± 18
Los Angeles	120 ± 12
Average	578
or	7.3 mc/mi²

(Probable additional Sr^{90} in lower layers and to be released by additional leaching probably will raise this about twofold.)

* Data by E. P. Hardy and R. S. Morse, of the Health and Safety Laboratory, New York Operations Office, personal communication.

† This datum was obtained by Dr. J. L. Kulp, Lamont Geological Observatory, Columbia University. The procedure was different from that of the New York Operations Office (personal communication).

TABLE 7

Sr^{90} CONTENT OF RIVER AND LAKE WATERS*

Location	Sr^{90} Content (dpm/Gal)	Location	Sr^{90} Content (dpm/Gal)
Lake Michigan, October 27, 1953	0.39 ± 0.08	Mosel River, Metz, September 7, 1953	0 ± 0.05
Mississippi River, Memphis, February 4, 1953	1.13 ± 0.16	Seine River, Nogent, September 8, 1953	0 ± 0.09
Mississippi River, St. Louis, April 17, 1953	<0.77 ± 0.18	Danube River, Ulm, September 12, 1953	0 ± 0.07

* See note to Table 2.

Examination of the data in Tables 2 and 3 on the Sr^{90} content of the exchangeable calcium in soils shows that there is a strong tendency for the lowest activity of Sr^{90} per unit weight of exchangeable Ca to occur in Ca-rich soils and vice versa, as would be expected, of course, if the fallout rate were uniform. This situation is displayed graphically for the pre-Castle soil data in Figure 6.

For years the Health and Safety Laboratory of the New York Operations Office of the Atomic Energy Commission (Mr. Merril Eisenbud, manager), with the co-

operation of the United States Weather Bureau, has been collecting fallout data[17] by use of gummed papers of 1-ft.^{2} area which are laid flat for a certain time out in the open away from buildings. After the exposure, the paper is folded and mailed to the New York Operations Office in an ordinary envelope. Samples thus can be collected cheaply, easily, and quickly from any populated area anywhere the postal service reaches. Most of the data so obtained have dealt with total fallout rather than with Sr^{90} specifically, but many analyses for Sr^{90} have been made since Operation Castle. These are presented later.

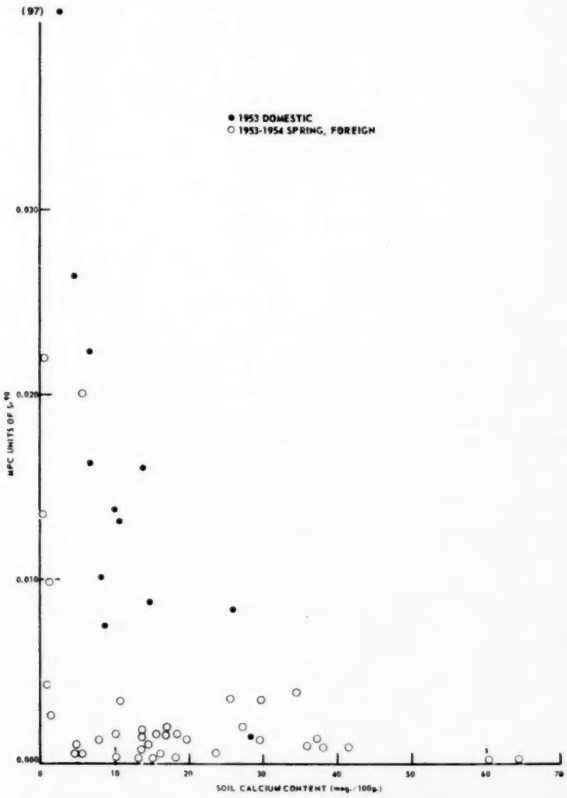

FIG. 6.—Top soil Sr^{90} concentration versus calcium content

The main question about the gummed-paper technique is its over-all efficiency of collection. In order to determine this, the Health and Safety Laboratory has conducted an extensive series of comparisons of the amounts of fallout by gummed paper and a 12-gallon pot with an 18-inch vertical cylindrical wall placed immediately beside the paper. Some of the data thus obtained are given in Table 8. From them we deduce a collection efficiency of 69 ± 9 per cent (but we use 63 per cent, since Mr. Eisenbud recommends this on the basis of more data and a better statistical treatment).

The data thus obtained for the post-Castle Sr^{90} fallout rate in the United States and South America are given in Table 9. These are combined with those for other areas, to give the world Sr^{90} fallout rates for September, October, November, and

TABLE 8

GUMMED PAPER COLLECTION EFFICIENCY
(Relative to 12-Gallon Pot [18-Inch Vertical Wall, 12-Inch Diameter; Cylindrical])

Month	Gummed Paper (dpm/Ft²/Mo)	Pot (dpm/Ft²/Mo)	Efficiency (Per Cent)	Reference
Mar., 1954	11.6	14	84	*
Apr., 1954	15.2	31	49	*
May, 1954	21.6	34	63	*
June, 1954	10.7	9.2	116	*
July, 1954	17.6	25	70	*
Aug., 1954	13.5	7.7	176	*
Sept., 1954	20.7	92	22	*
Oct., 1954	3.1	11	29	*
Nov., 1954	5.7	32	18	*
Jan., 1955	9.0	9.9	92	†
Feb., 1955	29.8	50.6	58	‡
Mar., 1955	210	150	140	‡
Apr., 1955	44.9	79.5	57	§
May, 1955	18.6	56.4	33	‖
June, 1955	12.4	51.7	24	‖
		Average	69 ± 9#	

* Interim Sunshine Report, NYOO Report NYO-4620, January 17, 1955.
† Sunshine Report for January and February, NYOO Report NYO-4643, April 21, 1955.
‡ Sunshine Report for March and April, NYOO Report NYO-4646, May 30, 1955.
§ Sunshine Report for May and June, NYOO Report NYO-4653, July 5, 1955.
‖ NYOO Report NYO-4623, January 18, 1955.
The figure of 63 per cent will be used for consistency. Mr. Merril Eisenbud (NYOO) recommends this on the basis of more data and a better statistical treatment.

TABLE 9

POST-CASTLE FALLOUT IN U.S. FROM GUMMED PAPERS
(Taken at 63 Per Cent Efficiency [cf. Table 8])

Month	Location	Sr^{90} Fallout Rate (Mc/Mi²/Yr)	Reference
Sept., 1954	Eastern U.S. (10 stations)	2.5 ± 0.2	*
Oct., 1954	Eastern U.S. (10 stations)	2.0 ± 0.4	*
Nov., 1954	Eastern U.S. (10 stations)	2.0 ± 0.4	*
Dec., 1954	Eastern U.S. (10 stations)	1.4 ± 0.2	†
Jan., 1955	Eastern U.S. (9 stations)	1.3 ± 0.2	†
Sept., 1954	Western U.S. (20 stations)	0.9 ± 0.2	‡
Sept., 1954	U.S. (38 stations)	0.92 ± 0.2	§
Oct., 1954	U.S. (38 stations)	0.79 ± 0.2	§
Nov., 1954	U.S. (38 stations)	0.95 ± 0.2	§
Dec., 1954	U.S. (37 stations)	0.71 ± 0.2	‖
Sept., 1954	South America (12 stations)	2.1 ± 0.2	‡
Oct., 1954	South America (12 stations)	1.6 ± 0.2	‡
Nov., 1954	South America (12 stations)	2.4 ± 0.2	‡

* Sunshine Report for March and April, NYOO Report NYO-4646, May 30, 1955.
† Sunshine Report for January and February, NYOO Report NYO-4643, April 21, 1955.
‡ Sunshine Report for May and June, NYOO Report NYO-4653, July 5, 1955.
§ Sunshine Report, August 1954, NYOO, HASL-8-2 (NYO-4656), November 12, 1954.
‖ Sunshine Report for July and August, NYOO Report NYO-4661, September 16, 1955.

December, 1954, presented in Table 10 and Figure 7. From these data we obtained these extremely important conclusions:

1. A Sr^{90} fallout probably derived from megaton weapons and nearly uniform over the world, except for local effects due to rainfall variations and to fallout from

submegaton weapons, seems clearly established. The fallout from the kiloton weapons lasts only a few weeks at most, since they involve only tropospheric

TABLE 10

WORLD-WIDE Sr^{90} FALLOUT RATE FROM GUMMED PAPERS*
(Taken at 63 Per Cent Efficiency [cf. Table 8]; Mc/Mi²/Yr)

Area	Per Cent Total Earth's Area	No. Stations (Sept.)	Sept., 1954	Oct., 1954	Nov., 1954	Dec., 1954	Average
Arctic	6.5	5	1.2 ± 0.4	3.0 ± 0.4	1.4 ± 0.4	0.9 ± 0.4	1.6 ± 0.2
North Temperate	10.9	14	1.7 ± 0.24	0.68 ± 0.24	1.4 ± 0.24	0.6 ± 0.24	1.1 ± 0.12
Pacific	8.0	2	0 ± 0.6	0.6 ± 0.6	1.9 ± 0.6	1.2 ± 0.6	0.9 ± 0.3
U.S.	1.5	39	1.3 ± 1.4	0.9 ± 0.14	1.4 ± 0.14	0.9 ± 0.14	1.1 ± 0.07
North Tropic	18.5	8	0.7 ± 0.3	1.5 ± 0.3	0.7 ± 0.3	0.4 ± 0.3	0.8 ± 0.16
South Tropic	25.5	9	3.2 ± 0.3	2.4 ± 0.3	2.5 ± 0.3	0.6 ± 0.3	2.1 ± 0.15
					Average (area weighted)		1.5 ± 0.1

Area	Jan., 1955	Feb., 1955	Mar., 1955	Apr., 1955	May, 1955	June, 1955	July, 1955	Aug., 1955	Average
Arctic	0.32	1.2	0.52	0.62	2.0	1.5	1.6	0.82	1.07
North Temperate	1.0	0.55	1.1	1.1	2.9	1.9	2.5	1.4	1.57
Pacific (21 stations)	0.53	0.58	2.2	1.1	1.6	1.3	1.2	1.0	1.19
U.S.	0.86	0.86	1.4	2.3	4.0	2.8	2.9	1.4	2.07
North Tropic	0.71	0.70	1.2	0.78	1.5	0.80	0.87	0.50	0.88
South Tropic	1.5	1.3	0.83	0.40	1.0	1.5	0.60	0.72	0.98
South Temperate (4 stations)	1.1	0.46	0.9	0.40	0.38	1.9	1.1	1.4	0.74
									0.95 ± 0.1

Average (area weighted, except U.S. and North Temperate omitted because of Teapot)

* Sunshine Report for July and August, NYOO Report NYO-4661, September 16, 1955, and personal communication from Dr. E. C. Plesset, Rand Corporation.

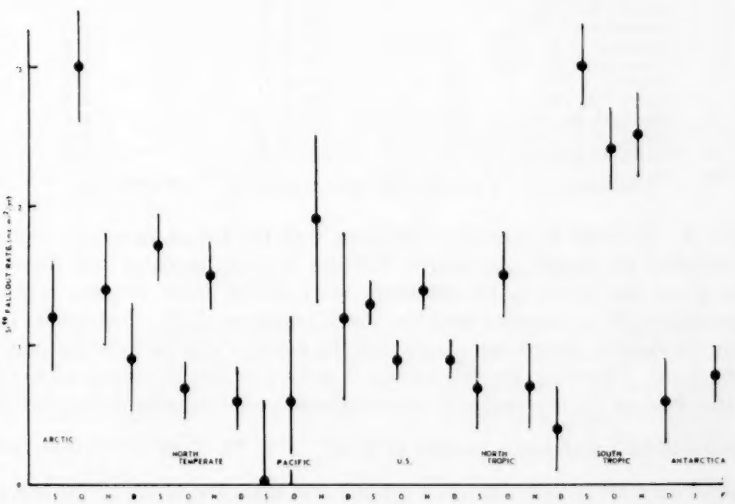

FIG. 7.—World-wide Sr^{90} fallout rates, September–December, 1954, (gummed paper at 63 per cent efficiency except antarctic value, which was snow).

storage, but widespread fallout is found to occur at least 1.7 years after a megaton test series.

2. This average world-wide Sr^{90} fallout rate in the fall of 1954 and the spring and summer of 1955 was 1.2 mc/mi²/yr (see Table 10).

Additional information available on the Sr^{90} distribution has been obtained by air filters operated at sea level, principally by the Naval Research Laboratory, measured in the Chicago Sunshine Laboratory, and by the Health and Safety Laboratory.[18] The surface samples collected at Washington, D.C., by the Naval Research Laboratory and measured at Chicago[8-12] are given in Table 11 and

TABLE 11

AIR-FILTER DATA, WASHINGTON, D.C.

(Army Chemical Corps Type 5 Paper, 99 Per Cent Efficient Down to Few Tenths Micron; 75 Per Cent Efficient Down to 0.01 Micron)

Washington, D.C.	Sr^{90} (dpm/10^6 Ft³)	Equivalent* Fallout (Mc/Mi²/Yr)
April 5–8, 1953	18.6 ± 0.7	0.14
October 2–6, 1953	41.1 ± 3.0	0.3
October 6–9, 1953	30.5 ± 1.1	0.2
October 12–15, 1953	70.4 ± 12	0.5
April 3–5, 1954	91.0 ± 7	0.7
April 8–10, 1954	6.4 ± 0.2	0.05
April 10–12, 1954	258 ± 6	1.9
April 12–14, 1954	65.5 ± 4.6	0.5
April 15–17, 1954	11 ± 0.5	0.08
April 17–19, 1954	21 ± 0.6	0.16
April 29–May 1, 1954	32.2 ± 2.6	0.2
May 11–13, 1954	31.3 ± 2.2	0.2
May 24–26, 1954	216 ± 11	1.6
June 1–3, 1954	68.3 ± 4	0.5
July 16–17, 1954	73.5 ± 5.2	0.5
July 26–29, 1954	48 ± 3.9	0.36
November 1–3, 1954	120 ± 7	0.9
December 1–2, 1954	103 ± 4	0.8
January 3–4, 1955	281 ± 6	2.1
February 5–6, 1955	127 ± 5	0.9
February 10–12, 1955	241 ± 10	1.8
February 22–23, 1955	202 ± 11	1.5
March 3–4, 1955	270 ± 13	2.0
March 7–8, 1955	394 ± 20	2.9
March 13–14, 1955	267 ± 16	2.0
March 16–17, 1955	310 ± 15	2.3
March 22–23, 1955	393 ± 20	2.9

* 134 dpm/10^6 ft³ = 1 mc/mi²/yr (cf. text); (28300 ft³/ft² = 10^6 ft³/35.5 ft³).

Figure 8. In order to correlate these data with the fallout rate, we recall that, as remarked previously, any rain of 0.1 inch or more probably will thoroughly wash down the fallout in the air below the layer at which the rain originates. Examination of the weather data for the Washington, D.C., area in the period when the samples were taken shows that the average interval between rains was 6 ± 3 days. Therefore, the Sr^{90} content of surface air should correspond to fallout for this time on the average, and we would expect a fallout rate R (mc/mi²/yr) to correspond to a surface-air content of $R \times \frac{6}{365} \times 79 \times 41 \times 2.5$ dpm/10^6 ft³, since 79 dpm/ft² is equivalent to 1 mc/mi² and there are 41 ft²/10^6 ft³ below the tropopause on the average at Washington, D.C., and 2.5 is the average ratio of height of tropopause to rain-bearing layers. The resulting rate of 0.70 ± 0.2

mc/mi²/yr is definitely low, compared to the rain result of 2.3 ± 0.2 mc/mi²/yr for Chicago (Fig. 5) and of 1.5 ± 0.1 mc/mi²/yr for data involving gummed paper (Fig. 7 and Table 9). The uncertainty in the factor of 2.5 for the ratio of rain-bearing layer height is probably the principal uncertainty in the calculation of the fallout rate from air-filter data, though it may be that the perfect vertical mixing of the lower 40 per cent of the troposphere implicit in the calculation is an incorrect assumption, in that air right at the surface is cleaned to a considerable extent by surface contact with vegetation and water and soil and therefore has less fallout than the average for the lower troposphere.

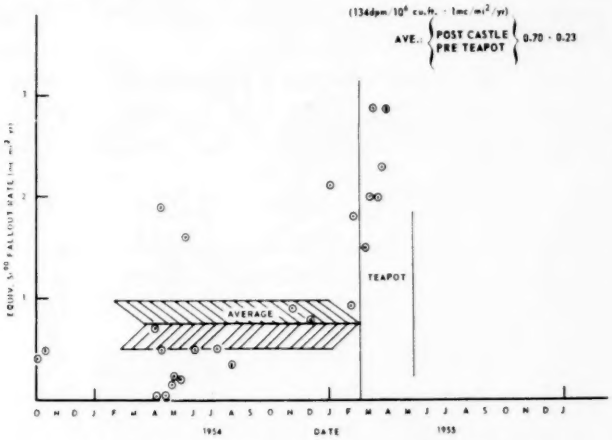

FIG. 8.—Sr^{90} in air, Washington, D.C.

From the data given on rates of fallout, we calculate the average stratospheric residence time, $\tau = 10 \pm 5$ years. The high-altitude data show a definite rise above the tropopause. This is strong confirmation for the stratospheric storage and dissemination mechanism.

II. DISCUSSION

A. PREDICTED Sr^{90} FALLOUT

The stratosphere reservoir of Sr^{90} immediately after Operation Castle had been completed was about 12 mc/mi². The fallout rate of Sr^{90} corresponds to an average storage time of 10 ± 5 years and essentially uniform world-wide dissemination. The radioactive half-life of Sr^{90} is 28 years, corresponding to an average life of 40 years. Therefore, the Sr^{90} fallout rate from tests up to and through the Castle series should be given by

$$R = \left(\frac{1}{10}\right) 12e^{-t(1/10+1/40)} \text{ mc/mi}^2\text{/yr}, \qquad (1)$$

where t is the time elapsed, in years, since May, 1954. This relation is given in Figure 9. From it, we can predict the tropospheric air content to be

$$A = \frac{RT \times 2.5 \times 79 \times 41}{365} \text{ dpm}/10^6 \text{ ft}^3, \quad (2)$$

where a period of T days elapses between the rains washing out the air mass concerned. Taking T to be 6 days on the average, the surface air should contain 134 R dpm/10^6 ft^3 in the middle latitudes.

The rainfall content will be 4 R dpm/gal for regions with an annual rainfall of 31.5 inches. The concentrations of Sr^{90} per unit volume for other annual precipitations are to be derived by inverse proportionately, e.g., in Antarctica, where the annual precipitation is about one-fourth of 31.5 inches, the Sr^{90} content of snow should be given by 16 R, or

$$A' = 16 \times 1.20e^{-t(1/10+1/40)} \text{ dpm/gal}. \quad (3)$$

In this connection, it is very important to note that regions of frequent rainfall very probably will receive more Sr^{90} fallout than will more arid regions.

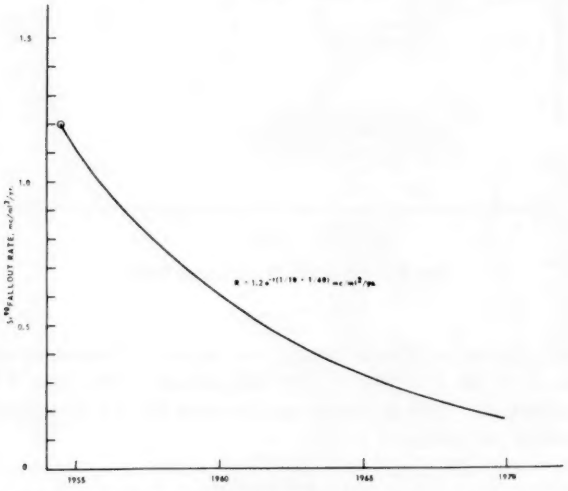

FIG. 9.—Predicted fallout rate for Sr^{90} formed prior to and during operation castle.

Of course, some fallout will be deposited by the surface winds blowing over the leaves of trees and grass. For example, the Naval Research Laboratory[10] has mounted an uncharged platinum screen vertically and held normal to the surface winds by a large vane. The deposition is by impact. Two weeks' total collections were made, and these gave up to 20 times as much as for gummed papers of the same area exposed for the same time in the same place. The screen was 80 mesh and probably passed about 0.5 × 10^6 ft.^{3} in the 2 weeks' exposure. From this it is clear that surface contact with fallout-laden tropospheric air must result in dep-

osition. Further evidence for this is seen in Table 1, where there is seen to be essentially no correlation between the Sr^{90} contents of soils and the crops of alfalfa grown on them. There obviously must have been very considerable direct dep osition on the surface of the plants. Also, the relatively low values for the Sr^{90} content of surface air found by the Naval Research Laboratory (Table 11), calculated with respect to the observed rain content, may very well be due to surface deposition by direct contact on tree leaves and grass.

The soil content will be the total of all fallout radioactivity, *less any natural weathering processes which serve to remove the fallout Sr^{90} from the chemically available form in which plants can assimilate it.* Neglecting this latter effect, although, as we shall see, there is reason to believe that such effects are operative in an important way, and taking the average for the exchangeable Ca content of soils over the world (cf. Tables 2 and 3), which is 8 gm/ft²/in. we calculate that the top 2.5 inches of soil, which in general holds nearly all the Sr^{90} (Tables 2 and 3), has some 20 gm of exchangeable Ca. Therefore, we predict that in the absence of curative weathering effects acting to remove Sr^{90} from contact with the biosphere, the average Sr^{90} concentration of exchangeable Ca, in MPC units, should be

$$S = \frac{0.0792}{2.2 \times 20}\left[P + \int_0^t R\,dt\right], \tag{4}$$

where P is the pre-Castle deposition of 0.8 mc/mi² in the middle latitudes and 0.2 mc/mi² world wide. Thus in the middle latitudes the Sr^{90} concentration of exchangeable soil Ca should be given in MPC units by

$$S = \frac{0.0792}{2.2 \times 20}\left[0.8 + 1.2\int_0^t e^{-t/10}\,dt\right]e^{-t/40} \tag{5}$$

or,

$$S = [0.0015 + 0.0213(1 - e^{-t/10})]e^{-t/40}. \tag{5'}$$

This result in terms of mc/mi² and MPC units is presented graphically in Figure 10. The maximum post-Castle Sr^{90} soil activity will be expected in about 1970. The present average should be about 0.005 MPC unit for soil with 20 gm. exchangeable Ca/ft². Those soils of low Ca content can, of course, have a much higher Sr^{90} content for unit amount of exchangeable Ca. Consider, for example, certain areas in Wales near Cardigan (cf. Table 3, Sample No. 54417), where the available Ca/ft² amounted to only 0.4 gm. and the specific Sr^{90} activity was found to be 0.097 MPC. For this area our analysis would predict a forty fold higher content than that given in Figure 10. This, of course, is reflected in higher contents for the bones of grazing animals.

The weathering processes which may operate to fix the fallout Sr^{90} and make it unavailable to the biosphere, such as the fixation in massive Ca deposits, are worthy of consideration. Only further investigation will reveal how important such processes are. The present Project Sunshine sampling program includes repeated sampling of given regions. The data so obtained should disclose any such trends. Some data already in hand seem to indicate such effects, but further confirmation is necessary.

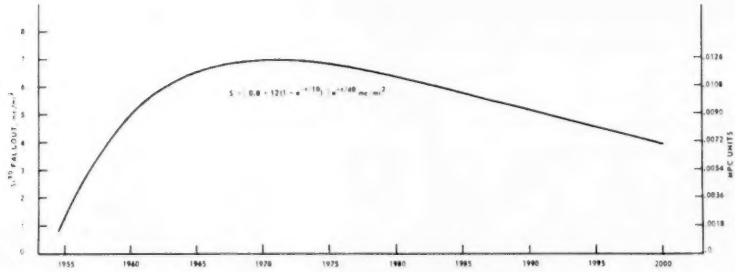

FIG. 10.—Predicted Sr^{90} in soil following Castle (Assumed Ca = 20 gms./ft²/2.5″)

In addition, palliative measures may prove effective. For example, Nervik, Kalkstein, and Libby[19] have shown that milk can be purified for radiostrontium by a treatment which may well prove to be quite practical and inexpensive.

B. BIOSPHERE CONTENT OF Sr^{90}

It seems clear that there will be discriminatory barriers of some magnitude operating at each of the stages of transfer from the soil into the biosphere. Taking these stages in general to be

(1) Soil → Plants,
(2) Plants → Animals,
(3) Animals (milk) → Humans,

we consider the three corresponding barriers.

1. *Soil-to-Plant Transfers.*—In Table 12 the Sr^{90} contents of plants are compared with those for the soils on which they grew at a variety of localities just before Operation Castle. It seems clear that, on the average, plants have about twice the specific Sr^{90} content relative to Ca that the supporting soils do. It seems

TABLE 12

PRE-CASTLE PLANT Sr^{90} CONTENTS
(Values Are Given in Terms of 1/1,000 MPC Units)

Location	Plant	Date	Reference	Plant Content	Content of Soil Which Grew Plant
U.S.	Alfalfa	(Average of Table 1)		8.9	4.7 ± 0.4
Turkey	Alfalfa	10–53	*	2.16 ± 0.18	1.2 ± 0.1
Cuba	Tobacco	4–54	*	1.7 ± 0.2	(1.6)†
New Zealand	Forage	11–53	‡	1.17 ± 0.28	0.21
New Zealand	Forage	1–54	‡	0.84 ± 0.10	0.18
Chile	Forage	11–53	§	0.84 ± 0.02	0.33*

* E. A. Martell and W. F. Libby, Project Sunshine Bull. No. 10, January 10, 1955.

† Calculated from average pre-Castle deposition (Fig. 4*a*) for latitude of Cuba and assumed average Ca content of 20 gm/ft² in top 2.5 inches.

‡ E. A. Martell, Project Sunshine Bull. No. 10, Suppl. 1, March 1, 1955.

§ E. A. Martell, *ibid.*, Suppl. 2, June 1, 1955.

very likely that this is due in part to fallout occurring directly on the external plant surfaces. This is further borne out by the lack of detailed correlation in Table 1 between soil and alfalfa results for eleven middle western United States farms. From this result, we may calculate that about half the total Sr^{90} content of alfalfa is due to direct fallout, while for general forage it is probably an even

higher percentage. This comparison is not quite accurate, because, although almost all the Sr^{90} is contained in the top 2.5 inches of soil, the Ca, on the other hand, is available to the entire depth of the root system. It is also recognized that the vertical distribution of the Ca may be nonuniform. There is no reason to expect preferential assimilation of Sr from the soil relative to Ca; hence the only other explanation for the data in Table 12 is direct fallout. As remarked in Section I in the paragraphs on rain and air-filter data, there are two mechanisms for direct fallout of the ultra-finely divided particulate matter carrying the Sr^{90} in the stratospheric reservoir—rainout and contact deposition after the fallout has entered the troposphere. Rainout appears to be the principal mechanism, though it has been demonstrated by the Naval Research Laboratory,[20] as mentioned earlier, that an 80-mesh screen mounted vertically to prevailing surface winds can gather more fallout than falls out directly on the average by all mechanisms. It is not clear, however, to what degree foliage acts in this way, but the probability seems high that the effect is relatively minor and that rainout, followed by drying of raindrops, is the main way in which the foliage surfaces gather fallout.

On this basis we conclude that total fallout in arid regions should be appreciably lower than in areas with normal rainfall. This effect seems to be borne out by the data available now, though more definitive experiments are needed. It probably follows also that regions subject to seasonal rainfall rather than relatively uniform precipitation all year should show less fallout for the same total annual rainfall. Also, regions subject to frequent morning fogs may be particularly high in total fallout. The Sr^{90} probably will enter the troposphere at relatively uniform rates, but the chance of precipitation will depend strongly on the local weather.

Of course, rainfall is necessary to plant growths, so that plants are certain to gather some fallout. However, for regions of low rainfall where irrigation is used—such as the Imperial Valley in California—the fallout content of the crops should be particularly low, for, as shown in Table 7, rivers are nearly free of fallout, since the soil purifies the runoff water before it reaches the rivers. Similarly, reservoirs and lakes will be low relative to rain because of dilution and the importance of runoff water from surrounding watersheds in replacing evaporative and withdrawal losses. It is also well to note that the ordinary water purification processes are effective in removing an appreciable fraction of the radiostrontium.

The by-passing of soil entirely, which occurs in the direct fallout on plant surfaces, of course means that the retarding effects of high Ca contents in soil are inoperative, and cattle grazing on such foliage may show little correlation in the Sr^{90} contents with the soil Sr^{90} activity for this reason. This appears, from the data in Table 1, to be true in the United States Midwest.

It is clear, however, that washing may reduce the level of fallout externally carried by plants, though direct leaf absorption will be expected to occur rather rapidly for water-soluble fallout. For the megaton weapons fired in the Pacific, the bulk of the fallout resides on particles of CaO or $Ca(OH)_2$ or mixtures of CaO, $Ca(OH)_2$, and $CaCO_3$ made by the great heat of the fireball acting on the coral of the islands and sea floor in the firing areas.[21] A large amount also is carried on NaCl particles. This material, therefore, should be quite water-soluble and should

be rapidly absorbed into the leaves. Washing therefore probably will not be particularly effective for the world-wide fallout, which derives from the Pacific tests. From weapons fired in the air, the particles probably will consist of less soluble oxides and therefore are more likely to wash off of plant surfaces before being absorbed.

Menzel,[22] of the United States Department of Agriculture, grew cowpeas on 42 American soils to which equal amounts of bomb debris had been added. Available Ca ranged from 0.7 to 48 milliequivalents of Ca/100 gm of soil. The Sr^{90}/Ca ratio of the plants was approximately inversely proportional to the available Ca in the soil over the full range of Ca availability. In another set of experiments on a particular type of soil (Evesboro) to which known amounts of Sr^{90} had been added at two carrier levels, the results listed in Table 13 were found. The distribution

TABLE 13

PLANT ASSIMILATION OF Sr^{90} FROM SOIL

Crop	(Sr/Ca) Soil (By Equivalents)	k_{Sr}	k_{Ba}
Barley	0.017	0.45	0.020
	0.0017	0.39	0.022
Buckwheat	0.017	0.49	0.023
	0.0017	0.43	0.028
Cowpeas	0.017	0.53	0.057
	0.0017	0.37	0.053

factor, k_{Sr}, defined as (Sr/Ca)plant/(Sr/Ca)soil, indicates the discrimination which the plant makes between Sr and Ca uptake. Similar tests were made for Ba.

By combining these data, Menzel concluded, as shown in Table 13, that the average Sr uptake from American soils was best fitted by a distribution factor of $k_{Sr} = 0.36$. This average probably will apply world-wide about as well.

2. *Plant-to-Animal Transfer.*—The Sr^{90} contained in grass and foliage eaten by grazing animals will be retained to an extent dependent on the metabolism of the animal. For example, for a 1-year-old steer[23] 30 per cent of the Sr^{90} fed orally was retained with essentially no discrimination relative to Ca. There appears to be a higher retention, approaching 90 per cent, for intravenous injection of young rats.[23] High Ca diets reduce the Sr^{90} uptake for rats, adult rats take up about 16 per cent of the ingested Sr^{90} on the same low Ca diet for which the young rats took up 73 per cent.[23]

Comar[24] has performed experiments on cows in which the Sr/Ca ratio in feed, blood, and milk was measured under equilibrium conditions. Typical relative values were 1.0, 0.37, and 0.13, respectively, thus indicating a relative lowering of the Sr^{90} content of the milk relative to the feed. This is borne out by direct observation on the fallout, as seen in Table 1, where for the Chicago milkshed the milk averaged 0.0014 MPC, while the alfalfa fed averaged 0.0089 MPC.

3. *Milk-to-Human Transfer.*—This ability of the cow to reduce the Sr^{90} in the milk relative to the feed is important as a barrier to human ingestion of this fallout radioactivity. With it in mind, we can expect that human Sr^{90} burdens should be 20 per cent or less (for older people) of the plant contents and about equal to the milk and cheese levels if the entire Ca in the body were assimilated at a given Sr^{90} content of the milk.

Since the Sr^{90} content of the whole biosphere is continually rising, about as shown in Figure 10, the average Sr^{90} content of milk should rise in a similar manner. Therefore, the intake of Sr^{90} by humans is steadily increasing, as shown in Figures 1*a*, 1*b*, 1*c*, and 1*d*. The data in Table 3 and Figures 1*c* and 1*d* show that the milk level at the time of Operation Castle averaged about 0.001 MPC, peaking in the middle latitudes, as did the soil assays (cf. Fig. 4*a*). This value shows a ratio of about 0.7 for the milk level to the average soil level for the top 2.5 inches of soil with 20 gm. of contained available Ca/ft^2. This high value probably is due to leaf pickup of fallout. Taking this as a general result, we predict the world values for milk and cheese at about 70 per cent of the soil values given in Figure 10, plus any local fallout. This would mean that we would expect average foreign milk and cheese samples to show about 0.004 MPC at the present time.

The human bone, of course, is formed during the growing process, so that the Sr^{90} content of children should be higher than for adults. The data[2, 8–12, 25] verify this prediction; however, for newborn babies there is less Sr^{90} than corresponds to the milk, showing the retarding effect of the mother's older Ca pool. Children seem to carry Sr^{90} approximately equal to the average level of the milk during the period of their lives. Adults decrease in Sr^{90} concentration with age as expected. It seems that the adults of the future will have Sr^{90} levels corresponding to the milk levels during their lifetime weighted according to their rate of growth. For example, the years from 10 to 18 will be most important for men and from 6 to 12 for women. Foreign children born now, according to Figure 10, should develop about 0.011 MPC unit during their lives. Children born now in the United States will develop a somewhat higher level, due to somewhat higher milk levels.

[1] "World-wide Effects of Atomic Weapons: Project Sunshine," Rand Corporation, R-251-AEC, August 6, 1953.

[2] W. R. Eckelmann and J. L. Kulp, Project Sunshine Bulletin, October 15, 1955.

[3] S. Kaufmann and W. F. Libby, "The Natural Distribution of Tritium," *Phys. Rev.*, **93,** 1337, 1954.

[4] H. von Buttlar and W. F. Libby, "Natural Distribution of Cosmic Ray Produced Tritium. II," *J. Inorg. and Nuclear Chem.*, **1,** 75, 1955.

[5] L. A. Currie, W. F. Libby, and R. Wolfgang, "Tritium Production by High Energy Protons," *Phys. Rev.*, **101,** 1557, 1956.

[6] F. Begemann and W. F. Libby, "Tritium Content of Surface Water, March 1954 to January 1956" (in press).

[7] Personal communication from Dr. Lester Machta, United States Weather Bureau.

[8] E. A. Martell and W. F. Libby, Project Sunshine Bulletin No. 10, January 10, 1955.

[9] E. A. Martell, Project Sunshine Bulletin No. 10, Suppl. 1, March 1, 1955.

[10] E. A. Martell, *ibid.*, Suppl. 2, June 1, 1955.

[11] E. A. Martell, *ibid.*, Suppl. 3, September 1, 1955.

[12] E. A. Martell, Project Sunshine Bulletin No. 11, December 1, 1955.

[13] G. H. Wilkening, "Natural Radioactivity as a Tracer in the Sorting of Aerosols According to Mobility," *Rev. Sci. Instr.*, **23,** 13, 1952.

[14] I. H. Blifford, L. B. Lockhart, Jr., and H. B. Rosenstock, "On the Natural Radioactivity in the Air," *J. Geophys. Research*, **57,** 499, 1952.

[15] O. Haxel and G. Schumann, "Selbstreinigung der Atmosphäre," *Z. Physik*, **142,** 127–132, 1955.

[16] P. E. Damon and P. K. Kuroda, "On the Natural Radioactivity of Rainfall," *Trans. Am. Geophys. Union*, **35,** 208, 1954.

[17] "World-wide Fallout from Operation Castle," NYOO Report NYO-4621, January 21, 1955.

[18] Project Sunshine: Progress from September 1953 to January 4, 1954, NYOO Report NYO-4571, January 8, 1954.

[19] W. E. Nervik, M. I. Kalkstein, and W. F. Libby, "Purification of Milk for Calcium and Strontium with Dowex X-50W Resin," University of California Radiation Laboratory Report No. UCRL-2674, August 12, 1954.

[20] I. H. Blifford, Jr., and H. B. Rosenstock, "Fallout Dosages at Washington, D.C.," Naval Research Laboratory Report NRL-4654, November 3, 1955; I. H. Blifford, Jr., L. B. Lockhart Jr., and R. A. Baus, "Relationship between the Air Concentration of Radioactive Fission Products and Fallout," Naval Research Laboratory Report NRL-4607, November 4, 1955; R. A. Baus, I. H. Blifford, Jr., and L. B. Lockhart, Jr., "Radioactivity of the Air," Naval Research Laboratory Report NRL-4509, March, 1955.

[21] "Fallout Symposium," AFSWP-895, January, 1955.

[22] R. G. Menzel and I. C. Brown, "Leaching of Fallout and Plant Uptake of Fallout," bimonthly report from United States Department of Agriculture, March–April, 1953.

[23] United States Atomic Energy Commission, Division of Biology and Medicine Report, No. 100A July, 1954.

[24] C. Comar, "Agricultural Research Program—Semiannual Progress Report for July 1–December 31, 1953," Report ORO-110, April, 1954, and C. L. Comar and R. H. Wasserman, "Radioisotopes in the Study of Mineral Metabolism," in *Progress in Nuclear Energy*, Vol. **5**, ("Biological Sciences and Medicine" [London: Pergamon Press, Ltd., 1956]) (in press).

[25] H. L. Volchok and J. L. Kulp, Project Sunshine Annual Progress Report, March 15, 1955.

[26] Sunshine Report for May and June, NYOO Report NYO-4653, July 5, 1955.

[27] Sunshine Report for July and August, NYOO Report NYO-4661, September 16, 1955.

[28] Sunshine Report, August, 1954, NYOO, HASL-S-2 (NYO-4656), November 12, 1954.

[29] NYOO Report NYO-4623, January 18, 1955.

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